Generalisation of two foil method for joint estimation of electron temperature and impurity content

J. Svoboda[1], M. Imríšek[1], J. Seidl[1], M. Tomeš[1]

[1] *Institute of Plasma Physics of the CAS, Prague, Czech Republic*

This contribution presents a probabilistic formulation of a generalised two-foil method. It is based on modelling soft x-ray radiation to determine plasma parameters (electron temperature and impurity content) from ratios of intensity measurements with different spectral filters.

The radiation model can include multiple impurities, however, this contribution focuses on tungsten as a first step. This is motivated by the prominent role of tungsten in fusion devices and by the intended application at COMPASS-Upgrade, a compact tokamak with a high magnetic field and plasma current (5 T, 2 MA).

This contribution has two main sections. The first one describes the radiation model used to produce synthetic measurements, which consists of Bremsstrahlung and tungsten line radiation. The second presents the generalisation of the two-foil method, which includes line radiation in the radiation model.

Radiation Model

The radiation model used to produce synthetic measurements consists of Bremsstrahlung and tungsten line radiation. The recombination radiation is not included as it is negligible in the hot plasma [1]. Wavelengths and photon emissivity coefficients for creation of line models are obtained from ADAS database [2].

The Bremsstrahlung formula is based on [3]. The spectral emissivity $\varepsilon_{\mathrm{Bs},i}$ emitted by Maxwellian electrons with density n_e and temperature T_e interacting with ions of density n_i and charge Z_i per unit volume per steradian per unit wavelength λ is

$$\varepsilon_{\mathrm{Bs},i}(\lambda) = C_{\mathrm{Bs}} \frac{n_\mathrm{e}}{\lambda^2 \sqrt{T_\mathrm{e}}} n_i Z_i^2 \, \mathrm{g}_{\mathrm{ff}}(Z_i, T_\mathrm{e}, \lambda) \mathrm{e}^{-\frac{hc}{\lambda k_\mathrm{B} T_\mathrm{e}}} \,, \tag{1}$$

where c is the speed of light, h is Planck's constant, k_B is Boltzmann's constant, and $\mathrm{g}_{\mathrm{ff}}(Z_i, T_\mathrm{e}, \lambda)$ is the Maxwellian free-free Gaunt factor [4]. The term C_{Bs} combines physical constants and numerical factors [5].

The distribution of tungsten ionisation states is determined based on local electron temperature and density. Particle transport effects on the ionisation balance are ignored.

Using the zero transport assumption, the formula from [6] for excitation line could be rewritten, expressing the spectral emissivity of lines $\varepsilon_{\mathrm{Ln,W}^Z}$ emitted by tungsten ions with charge Z

as

$$\varepsilon_{\mathrm{Ln,W}^Z}(\lambda) = \frac{1}{4\pi} n_{\mathrm{W}} n_{\mathrm{e}} A_{\mathrm{W}^Z}(T_{\mathrm{e}}, n_{\mathrm{e}}) \sum_{l}^{N_Z} \mathrm{PEC}_l^{\mathrm{W}^Z}(T_{\mathrm{e}}, n_{\mathrm{e}}) f_l(\lambda)\,, \tag{2}$$

where n_{W} is tungsten density, $A_{\mathrm{W}^Z}(T_{\mathrm{e}}, n_{\mathrm{e}})$ is the fraction abundance of tungsten ions with charge Z, N_Z is the number of lines in spectra of this ion, $\mathrm{PEC}_l^{\mathrm{W}^Z}(T_{\mathrm{e}}, n_{\mathrm{e}})$ is the photon emission coefficient of line l, and $f_l(\lambda)$ is a normalised spectral line shape of line l.

The total spectral emissivity then can be expressed as a sum of Bremsstrahlung and line radiation contributions over all ion species present in the plasma. Additional impurity species can be incorporated into the radiation model by adding their corresponding line radiation terms similar to tungsten in (2).

To obtain the power recorded by the detector, the effects of the filter and diode sensitivity must be included. For detector volume sensitivity $G(\mathbf{r})$, where $\mathbf{r}$ is the position vector of a radiating point, the incident power recorded by the detector $P_{\mathrm{det}}^{(d)}$ fitted with a filter with spectral sensitivity $\mathit{eff}(d,\lambda)$ for a thickness d is

$$P_{\mathrm{det}}^{(d)} = \int \mathit{eff}(d,\lambda) \int G(\mathbf{r})\varepsilon(\lambda,\mathbf{r})\mathrm{d}\mathbf{r}\mathrm{d}\lambda\,. \tag{3}$$

Generalisation of the Two-Foil Method

The standard two-foil method neglects the line radiation contribution and assumes that the measured intensity is dominated by Bremsstrahlung radiation [7]. This assumption is violated in the case of metallic wall machines with substantial heavy impurity content in the plasma. To compensate for this, we propose a generalisation of the two-foil method that includes line radiation in the radiation model.

Assuming quasi-neutrality of plasma and substituting effective charge definition in (1) gives the effective Bremsstrahlung emissivity

$$\varepsilon_{\mathrm{eff}}(\lambda) = C_{\mathrm{Bs}} \frac{n_{\mathrm{e}}^2}{\lambda^2\sqrt{T_{\mathrm{e}}}} Z_{\mathrm{eff}}\, g_{\mathrm{ff}}(Z_{\mathrm{eff}}, T_{\mathrm{e}}, \lambda) \mathrm{e}^{-\frac{hc}{\lambda k_{\mathrm{B}} T_{\mathrm{e}}}}\,. \tag{4}$$

In order to derive the generalised ratio function, the detector effectivity must be accounted for. Applying detector sensitivity to radiation models for effective Bremsstrahlung (4) and for line radiation (2) gives

$$\mathscr{P}_{\mathrm{fwd}}^{(d)} = \int \mathit{eff}(d,\lambda)\varepsilon_{\mathrm{eff}}(\lambda)\mathrm{d}\lambda + \int \mathit{eff}(d,\lambda) \sum_{i\in\mathrm{W}} \varepsilon_{\mathrm{Ln},i}(\lambda)\mathrm{d}\lambda\,. \tag{5}$$

Let us define Bremsstrahlung and line radiation proportionality terms $a_{\mathrm{Bs}}^{(d)}$ and $a_{\mathrm{Ln}}^{(d)}$ as

$$a_{\mathrm{Bs}}^{(d)}(T_{\mathrm{e}}, Z_{\mathrm{eff}}) = C_{\mathrm{Bs}} Z_{\mathrm{eff}} \int \frac{\mathit{eff}(d,\lambda)}{\lambda^2\sqrt{T_{\mathrm{e}}}} g_{\mathrm{ff}}(Z_{\mathrm{eff}}, T_{\mathrm{e}}, \lambda) \mathrm{e}^{-\frac{hc}{\lambda k_{\mathrm{B}} T_{\mathrm{e}}}} \mathrm{d}\lambda\,. \tag{6}$$

and

$$a_{\mathrm{Ln}}^{(d)}(T_\mathrm{e},n_\mathrm{e}) = \sum_{Z=1}^{74} \frac{A_{\mathrm{W}^Z}(T_\mathrm{e},n_\mathrm{e})}{4\pi} \sum_{l}^{N_\mathrm{lines}} \mathrm{PEC}_l^{\mathrm{W}^Z}(T_\mathrm{e},n_\mathrm{e}) \int \mathit{eff}(d,\lambda) f_l(\lambda)\mathrm{d}\lambda\,. \tag{7}$$

Using these definitions gives

$$\mathscr{P}_{\mathrm{fwd}}^{(d)} = n_\mathrm{e}^2 a_{\mathrm{Bs}}^{(d)}(T_\mathrm{e},Z_\mathrm{eff}) + n_\mathrm{e} n_\mathrm{W} a_{\mathrm{Ln}}^{(d)}(T_\mathrm{e},n_\mathrm{e})\,. \tag{8}$$

The generalised ratio function R_g for two different filter thicknesses d_1 and d_2 is expressed as

$$R_\mathrm{g}^{(d_1,d_2)}(T_\mathrm{e},n_\mathrm{e},n_\mathrm{W},Z_\mathrm{eff}) = \frac{n_\mathrm{e} a_{\mathrm{Bs}}^{(d_1)}(T_\mathrm{e},Z_\mathrm{eff}) + n_\mathrm{W} a_{\mathrm{Ln}}^{(d_1)}(T_\mathrm{e},n_\mathrm{e})}{n_\mathrm{e} a_{\mathrm{Bs}}^{(d_2)}(T_\mathrm{e},Z_\mathrm{eff}) + n_\mathrm{W} a_{\mathrm{Ln}}^{(d_2)}(T_\mathrm{e},n_\mathrm{e})}\,. \tag{9}$$

This is the equation used in the probabilistic model implemented in PyMC [8]. It uses priors on electron temperature T_e, electron density n_e, tungsten density n_W, and effective charge Z_eff.

The behaviour of the ratio depends mostly on the electron temperature and tungsten density, while the electron density and effective charge have a smaller effect. Exploring the behaviour in the parameter space, three different regimes can be identified based on the sensitivity of the ratio to electron temperature and tungsten density: Bremsstrahlung dominated, impurity content dominated, and correlated. These regions are shown in figure 1 and the posteriors for three different scenarios in these regimes are shown in figure 2.

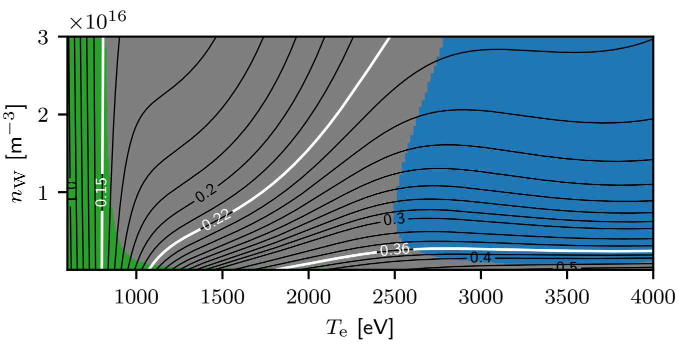

Figure 1: Behaviour of the generalised ratio function in the parameter space of electron temperature and tungsten density, with three distinct regimes: Bremsstrahlung dominated (green), impurity content dominated (blue), and correlated (gray).

Conclusions

A Bayesian model of the generalised ratio method was implemented using PyMC. Three distinct areas of behaviour in parameter space were identified based on sensitivity of the ratio to either electron temperature or tungsten density.

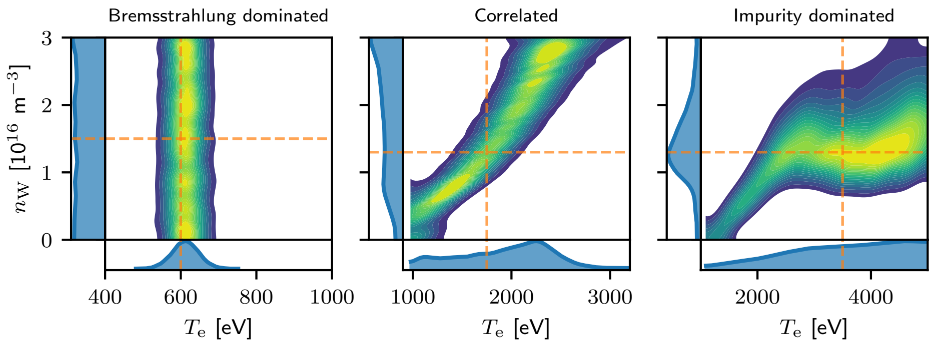

Figure 2: Posterior distributions of electron temperature and tungsten density for three different scenarios in the parameter space of electron temperature and tungsten density. The true values are marked with a orange dashed lines.

The method is designed and implemented in a way that allows extensibility to additional heavy impurities or more accurate atomic data. It is planned to include additional impurities into the model.

The results in the correlated regime could be improved by integration with additional diagnostics, such as Thomson scattering or interferometry. These can be used to limit the parameter space for the ratios. Another possibility is to use absolute calibration of the detectors. This way, the recorded power is matched, effectively limiting the parameter space to only part of the ratio contour. Alternatively, an additional filter could be used to obtain three ratios. The posteriors are more closely defined by the intersections of the three ratio contours.

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