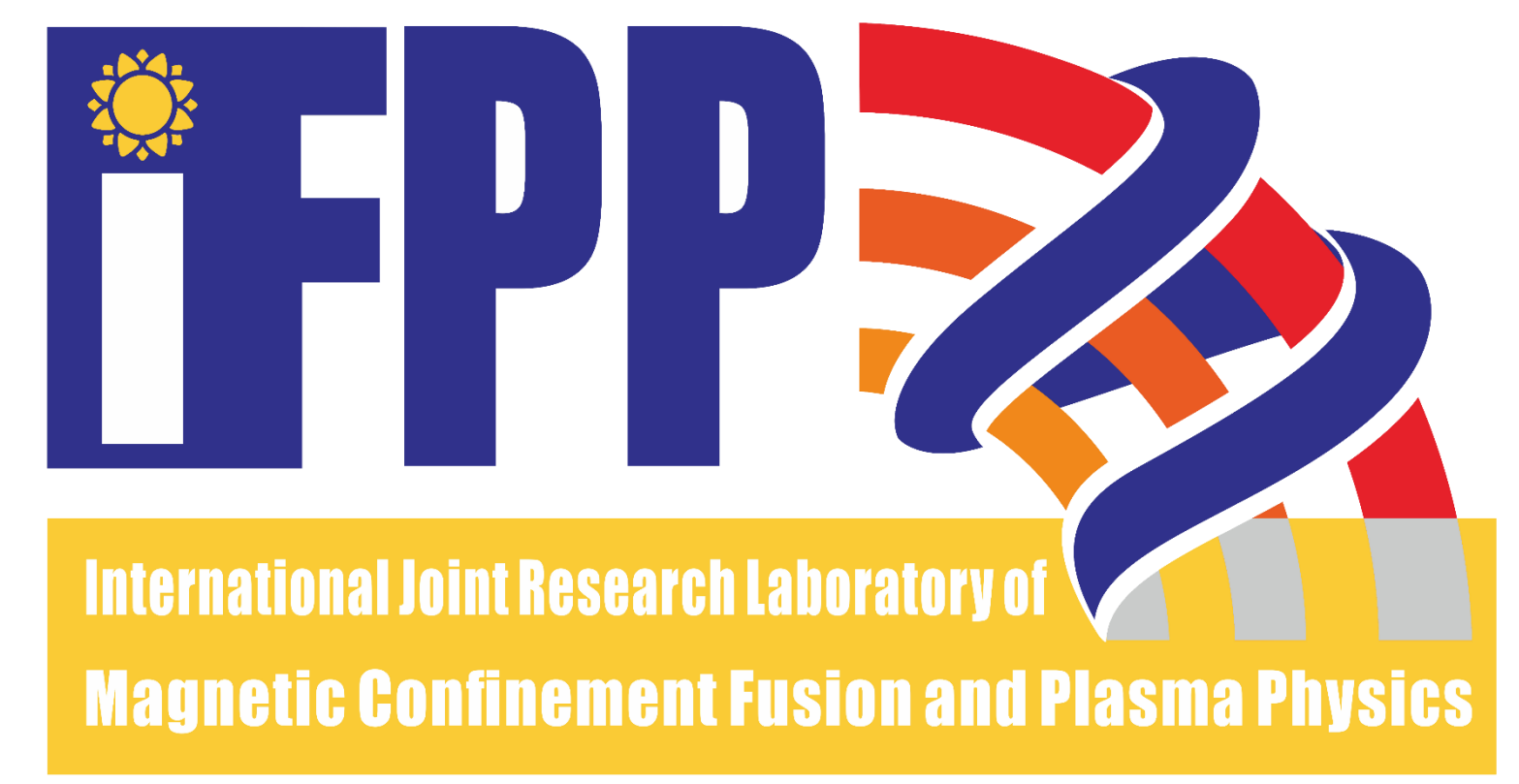


Theory of impurity effects on transport driven by micro-tearing mode (MTM) in the strong gradient pedestal of tokamak plasmas



Shanni Huang, Weixin Guo, Lu Wang

Huazhong University of Science and Technology, Wuhan, 430074

Impurity effects on MTM in the pedestal

Impurity effects → MTM instability → electron heat transport

- Collisionality $\nu_{ei} \uparrow \rightarrow \gamma_{MTM} \uparrow$ first \uparrow then \downarrow ; for impurity, $Z_{eff} \uparrow \rightarrow \nu_{ei} \uparrow$
- Gradient ratio $\eta_i \uparrow \rightarrow \gamma_{MTM} \downarrow$; stronger dilution effects for $\nabla n_z \rightarrow L_{ni} \downarrow \rightarrow \eta_i \uparrow$
- DIII-D pedestal: MTM occur → $P_e \downarrow$ → electron heat transport

Studying the impurity effects on MTM is helpful for understanding the electron heat transport

1/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

Ordering: based on pedestal parameters on DIII-D

DIII-D strong gradient pedestal parameters dominated by MTM

Pulse	q	T_e (keV)	$n_e(10^{20} \text{ m}^{-3})$	$\nu_{ei}(s^{-1})$
175823	5.59	0.197	3.57	7.38
0.98	4.94	37.0	17.1	7.24
R (m)	$a(m)$	B_0 (T)	ρ_s (mm)	c_s (km s ⁻¹)
1.73	0.739	1.977	1.023	69.1

Based on these parameters, orderings:

$$\frac{\omega}{\alpha_s} \sim \frac{\nu_{ei}}{\alpha_s} \sim \frac{1}{\beta_e} \gg 1$$

2/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

Perturbed density in slab configuration

Perturbed distribution function of electron: drift kinetic equation

$$(\omega - k_y v_y) \delta f_e - i C_{ei}(\delta f_e) = \tilde{\omega}_e \tau_{Te} \frac{e}{T_e} (\delta \phi - v_y \delta A) - i F_{\text{coll}} \frac{e}{T_e} v_y \delta E_1$$

➤ Lorentz gas (electron-ion) collision operator $C_{ei}(\delta f_e) = (1/2) \nu_{ei} (\partial/\partial v) (1 - v^2) (\partial/\partial v) \delta f_e$

➤ $\frac{\partial}{\partial v} (1 - v^2) \frac{\partial}{\partial v} P_e(v) = -n(1 + v^2) P_e(v) \rightarrow$ expand δf_e as $\delta f_e = \sum h_n(v) P_n(v)$

Perturbed parallel current density: $\delta j_{\parallel} = -e \int d^3v \delta f_e v_{\parallel} = -e \int d^3v h_1(v) v_{\parallel} P_1(v) = \sigma_e \delta E_1$

Perturbed electron density: $\delta n_e = \int d^3v \delta f_e = n_e \frac{\alpha_s}{\omega} \delta \phi - \frac{k_y}{\omega} \sigma_e \delta E_1$

➤ Impurity terms → parallel electron conductivity $\sigma_e \rightarrow \delta n_e$

Perturbed ion/impurity density: GK equation → $\delta n_i = -Z_i \tau_{Ti} \frac{e}{T_i} \left[\frac{\omega}{\omega} + \left(1 - \frac{\omega}{\omega} (1 + \eta_i) \right) \frac{e}{T_i} \right] \delta \phi$

$\delta n_e = \delta n_i + Z \delta n_z$ + parallel Ampere's law → EM eigen-equation with impurity

3/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

General solution of eigen-equation in the ion region $x \sim \rho_i$

Eigen-equation of $\hat{J}(k)$ when $\hat{k} \rightarrow 0$

$$\frac{d}{dk} \left[\frac{(1 - \hat{\omega}^2) + (\rho_{im}^2 / \rho_e^2 + K_{eff}) k^2}{(\rho_{im}^2 / \rho_e^2 + K_{eff}) k^2} \right] \frac{d\hat{J}}{dk} + \left(\frac{1}{4} - \mu^2 \right) \frac{d\hat{J}}{dk} = 0$$

with $\frac{1}{4} - \mu^2 = \frac{\hat{\omega}^2 \beta_e}{1 + Z_{eff} \tau_{Te}} \left(\frac{1}{\omega} + Z_{eff} \tau_{Te} \right) \left(1 - \frac{\omega}{\omega} \right) \left(\tau_{Te} = \tau_{Te}, Z_{eff} = Z_{eff}, \hat{\omega} = \omega / \alpha_s \right)$

➤ Boundary condition: matching to the MHD region

$$\hat{J}(k) \sim 1 - \frac{4\mu^2}{8} k^2 + \frac{\pi}{32\Delta\rho_e} \hat{\omega} \frac{\rho_{im}^2}{\rho_e^2 + K_{eff}} k^2 \rightarrow \frac{\alpha_s}{\omega} = \frac{17\hat{\omega}}{32\Delta\rho_e} \hat{\omega} \left(\frac{\rho_{im}^2}{\rho_e^2 + K_{eff}} \right) \quad [\text{Pegoraro, POF1989}]$$

For $\hat{k} \rightarrow \infty$, the general solution of $\hat{J}(k)$ can get

$$\hat{J}(k) \sim y^{1+\mu/2} \left[a \frac{\Gamma(-1/2)\Gamma(-\mu)}{\Gamma^2(-1/4-\mu/2)} - i a \frac{\Gamma(5/2)\Gamma(-\mu)}{\Gamma^2(5/4-\mu/2)} \right] + y^{1+\mu/2} \left[a \frac{\Gamma(-1/2)\Gamma(\mu)}{\Gamma^2(-1/4+\mu/2)} - i a \frac{\Gamma(5/2)\Gamma(\mu)}{\Gamma^2(5/4+\mu/2)} \right]$$

i.e. $\hat{J}(k) \sim \hat{\alpha}_s k^{1+2\mu} + \hat{\alpha}_s k^{3+2\mu}$

4/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

General solution of eigen-equation in the electron region $x \sim \delta$

Eigen function of $J(s)$ for $s = x/\delta \rightarrow \infty$

$$\frac{d^2}{ds^2} \left[\frac{(1 + \hat{\omega}^2 s^2 + \hat{\omega}^2 s^4)}{(\sigma_e + \sigma_e s^2)} \right] J(s) = -\hat{\beta}_e \hat{\omega}^2 J(s) \rightarrow s^2 \frac{d^2}{ds^2} \bar{J} = \left(-\frac{s^2}{2} - \mu^2 + \frac{1}{4} \right) \bar{J}$$

with $s^2 = -\hat{\beta}_e \hat{\omega}^2 \frac{\sigma_e}{\alpha_s} (v_e \gg \omega)$

➤ Boundary condition: for $s \rightarrow 0$, $J(s)$ is finite → $\bar{J} = \sqrt{\frac{s}{2}} K_{\nu} \left(\frac{s}{\sqrt{2}} \right)$

The general solution of $J(s)$ for $s \rightarrow \infty$: $J(s) \sim b_1 s^{-3/2-\mu} + b_2 s^{-3/2-\mu}$ with $b_1 = \left(\frac{s}{2} \right)^{3/2} \frac{\Gamma(1-\mu)}{\Gamma(1+\mu)}$

➤ Fourier transformation between $s \rightarrow t = \delta k$: $\hat{J}(k) \sim \hat{\alpha}_s k^{1+2\mu} + \hat{\alpha}_s k^{3+2\mu}$ with $\hat{\alpha}_s = \frac{b_1}{b_2} \frac{\Gamma(1-\mu)}{\Gamma(1+\mu)} \tan \left[\frac{\pi}{2} (1 + \mu) \right]$

5/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

Dispersion relation of MTM with impurity

Matching $\hat{J}(k)$ in ion region and $\hat{J}(t)$ in electron region: $\frac{\hat{\alpha}_s}{\alpha_s} = \frac{\hat{\alpha}_s}{\alpha_s} \left(e^{-i\omega t} \frac{\hat{\omega}^2 \delta_e}{\rho_e} \right)^{1/2}$

$$e^{i\omega t} R^{\mu} = \frac{\mu + \frac{1}{2}}{2} \frac{\Gamma(-\mu)}{\Gamma(-\mu)} \frac{D - \cot \left[\frac{\pi}{4} \left(\frac{\mu}{2} \right) \right]}{D - \cot \left[\frac{\pi}{4} \left(\frac{\mu}{2} \right) \right]}$$

with $R = 16 \frac{d_1}{1.71 \eta_e} \frac{\rho_{im}^2}{\rho_e^2} \frac{\rho_{im}^2}{\rho_e^2 + K_{eff}} + D = \frac{2}{\pi} \Delta \rho_e \frac{(\rho_{im}^2 / \rho_e^2)^{1/2}}{(\hat{\omega} - \nu)^{1/2}} \frac{\Gamma(5/4 - \mu/2) \Gamma(5/4 + \mu/2)}{\Gamma(3/4 - \mu/2) \Gamma(3/4 + \mu/2)}$

Assuming $\hat{\omega} \approx 1 + \delta \hat{\omega}$ and $\mu \approx 1/2 - \hat{\beta}_e \delta \hat{\omega}$, get the real frequency and growth rate

$$\delta \hat{\omega} = \frac{2}{\rho_{im}^2 / \rho_e^2 + K_{eff}} \frac{1}{\hat{\beta}_e \Delta \rho_e} \left(\frac{1.71 \eta_e}{\rho_e} \right)^{1/2} \frac{\delta_e}{\rho_e}$$

$$\hat{\gamma} = -4 \frac{1}{\hat{\beta}_e \Delta \rho_e} \frac{1}{(\rho_{im}^2 / \rho_e^2 + K_{eff})} \frac{\delta_e}{\rho_e} \left(\frac{1.71 \eta_e}{\rho_e} \right)^{1/2} \left(1 + \sqrt{2} \Delta \rho_e \hat{\beta}_e \frac{\delta_e}{\rho_e} \left(\frac{1.71 \eta_e}{\rho_e} \right)^{1/2} \right)$$

6/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

Dispersion relation of MTM with impurity

The real frequency and growth rate

$$\hat{\omega}_r = 1 + \frac{2}{\rho_{im}^2 / \rho_e^2 + K_{eff}} \frac{1}{\hat{\beta}_e \Delta \rho_e} \frac{1}{\rho_e} \frac{2\sqrt{2}}{\rho_e} \frac{1.71 \eta_e}{\rho_e} \frac{\delta_e}{\rho_e}$$

$$\hat{\gamma} = -4 \frac{1}{\hat{\beta}_e \Delta \rho_e} \frac{1}{(\rho_{im}^2 / \rho_e^2 + K_{eff})} \frac{\delta_e}{\rho_e} \left(\frac{1.71 \eta_e}{\rho_e} \right)^{1/2} \left(1 + \sqrt{2} \Delta \rho_e \hat{\beta}_e \frac{\delta_e}{\rho_e} \left(\frac{1.71 \eta_e}{\rho_e} \right)^{1/2} \right)$$

with $\rho_{im}^2 / \rho_e^2 = \sum \frac{m_i Z_i^2}{m_i Z_i} = 1$ (fully ionized light impurity), $K_{eff} = \sum \frac{m_i Z_i^2}{m_i Z_i} \frac{1}{Z_i} (1 + \eta_i)$, $\hat{\omega}_r^2 = \frac{2\nu_{ei}}{k^2 \tau_{Te}^2}$

➤ For MTM, $\Delta' < 0 \rightarrow \hat{\omega}_r, \hat{\gamma} > 0$, an unstable mode

➤ ν_{ei} is a drive factor of MTM, with $\nu_{ei} \uparrow, \hat{\gamma}$ first increases and then decreases

➤ Impurity effects: $\rho_{cesf}, K_{eff} \uparrow$, MTM is stabilized; $Z_{eff} \uparrow \rightarrow \nu_{ei} \uparrow$

➤ In high β_e plasma, MTM is stabilized

7/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

Electron heat transport flux driven by MTM

The electron energy flux driven by MTM with impurity $Q_e = \text{Re} \left(\int d^3v \frac{1}{2} m_e (v_{\parallel}^2 + v_{\perp}^2) \delta f_e \left(\mathbf{B} \times \frac{\nabla \delta A}{B^2} \right) \right)$

➤ The δf_e can be expressed as $\delta f_e = \chi_{eff}^e \delta \phi + \chi_{AA}^e \delta A_{\parallel} F_{\text{coll}}$

➤ $Q_e = -\sum \nu_{ei} n_e \nabla T_e |\delta \phi|^2 + \sum \nu_{ei} n_e \nu_{ei} |\delta \phi|^2 + \sum \nu_{ei} n_e \nu_{ei} |\delta \phi|^2 = Q_{coll} + 2Q_{AA} + Q_{eff}$

The energy flux also includes which induced by particle convection:

$$Q_e = \text{Re} \left(\int d^3v \frac{3}{2} T_e \delta f_e \left(\mathbf{B} \times \frac{\nabla \delta A - \nu_{ei} \delta A_{\parallel}}{B^2} \right) \right)$$

The net electron heat flux: $q_e = Q_e - Q_{eff}$, and effective electron heat conductivity:

$$\chi_{eff}^e = -\frac{q_e}{\nu_{ei} \nabla T_e} = \left[\frac{33}{4} \frac{\gamma_e}{(\omega_s + \omega_e)} + \frac{7\gamma_e}{(\omega_s + \omega_e)} \left(1 - \frac{\omega_s}{\omega_s + \omega_e} \right) \right] \frac{1}{\omega_s + \omega_e} \left(\frac{1}{\omega_s + \omega_e} \right) \left(\frac{2\omega_s}{\omega_s + \omega_e} \right) \left(\frac{1}{\omega_s + \omega_e} \right) \left(\frac{1}{\omega_s + \omega_e} \right) |\delta \phi|^2$$

8/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

Particle transport flux driven by MTM

The electron flux driven by MTM with impurity $\Gamma_e = \text{Re} \left(\int d^3v \delta f_e \left(\mathbf{B} \times \frac{\nabla \delta A - \nu_{ei} \delta A_{\parallel}}{B^2} \right) \right) = \Gamma_e^{coll} + \Gamma_e^{eff}$

➤ The effective electron diffusive transport coefficient

$$D_e^{eff} = \frac{\Gamma_e}{\nabla n_e} = (k_y \rho_e \epsilon)^2 \left[\frac{2\gamma_e}{(\omega_s + \omega_e)} + \frac{2\gamma_e}{(\omega_s + \omega_e)} \left(1 - \frac{\omega_s}{\omega_s + \omega_e} \right) \right] \frac{1}{(\omega_s + \omega_e)} \left[\frac{\gamma_e}{\omega_s + \omega_e} + \frac{3\gamma_e}{2(\omega_s + \omega_e)} \left(2 - \frac{\omega_s}{\omega_s + \omega_e} \right) \right] \frac{1}{k_y \rho_e \epsilon (\omega_s + \omega_e)} \left[\frac{\omega_s}{\omega_s + \omega_e} - \frac{\omega_s^2}{\omega_s + \omega_e} \right] |\delta \phi|^2$$

➤ D_e^{eff} is much smaller than χ_{eff}^e : $D_e^{eff} / \chi_{eff}^e \sim 0.1$ [Kotschenreuther, NF2019]

Non-Ambipolarity of MTM particle transport

➤ As $f_i = -Z_i \tau_{Ti} \frac{\alpha_s}{1 + \eta_i} \left[1 + \eta_i (v_{\parallel}^2 - 3)/2 \right] F_{\text{coll}} \delta \phi$, the ion/impurity flux: $\Gamma_i = - \left(\int d^3v \delta f_i \left(\mathbf{B} \times \frac{\nabla \delta A - \nu_{ei} \delta A_{\parallel}}{B^2} \right) \right) = \Gamma_i^{coll} + \Gamma_i^{eff}$

➤ $\Gamma_i^{coll} = - \sum \frac{Z_i^2}{(\omega_s + \omega_e)} \left(1 - \eta_i \right) \nabla n_i + \frac{b_z}{Z_i} k_y^2 \rho_e^2 \epsilon^2 |\delta \phi|^2$

➤ Quasi-neutrality condition → $\Gamma_i^{coll} + Z \Gamma_e^{coll} - \Gamma_i^{eff} - \Gamma_e^{eff} = 0$

➤ Γ_e^{eff} can not be compensated, i.e. the ambipolarity is not satisfied intrinsically for electromagnetic MTM turbulence

9/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

$\hat{\beta}_e$ on MTM instability

$\hat{\omega}_r = 1 + \frac{2}{\rho_{im}^2 / \rho_e^2 + K_{eff}} \frac{1}{\hat{\beta}_e \Delta \rho_e} \frac{1}{\rho_e} \frac{2\sqrt{2}}{\rho_e} \frac{1.71 \eta_e}{\rho_e} \frac{\delta_e}{\rho_e}$

with $\hat{\beta}_e = (\beta_e / 2) L_{ni}^2 / L_{ne}$, $K_{eff} = \sum \frac{m_i Z_i^2}{m_i Z_i} \frac{1}{Z_i} (1 + \eta_i) = f_{Le} (1 + \eta_e) + f_{Li} (1 + \eta_i)$, $C_{ei} = \left(\frac{1.71}{d_1} \right)^{1/2} \frac{1}{\sqrt{2}} \frac{1}{k_y \rho_e a} \frac{1}{\rho_e}$, $\alpha_s = \frac{\omega_s}{c_s} = k_y \rho_e \frac{a}{L_{ne}}$

$\hat{\beta}_e \uparrow$: $\beta_e \uparrow$ or $a/L_{ne} \uparrow$

When $f_c = 0$, $\hat{\beta}_e \uparrow \rightarrow \omega_r, \gamma_k \downarrow$: from 0.6 to 0.9, ω_r decreases about 59%, γ_k decreases about 58%

When $Z f_c = 0.2$, $A_z(Z) \uparrow \rightarrow \omega_r \downarrow$: $Z_{eff} \uparrow$ is more dominant than the $K_{eff} \downarrow$; for small $\hat{\beta}_e$, $A_z(Z) \uparrow \rightarrow \gamma_k \uparrow$, and opposite for large $\hat{\beta}_e$

10/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

$\hat{\beta}_e$ on electron heat conductivity

$\chi_{eff}^e = \left[\frac{33}{4} \frac{\gamma_e}{(\omega_s + \omega_e)} + \frac{7\gamma_e}{(\omega_s + \omega_e)} \left(1 - \frac{\omega_s}{\omega_s + \omega_e} \right) \right] \frac{1}{\omega_s + \omega_e} \left(\frac{1}{\omega_s + \omega_e} \right) \left(\frac{2\omega_s}{\omega_s + \omega_e} \right) \left(\frac{1}{\omega_s + \omega_e} \right) \left(\frac{1}{\omega_s + \omega_e} \right) |\delta \phi|^2$

χ_{AA}^{eff} is largest, χ_{eff}^e is about half of χ_{AA}^{eff}

$\beta_e \uparrow \rightarrow \chi_{AA}^{eff} \downarrow$: due to the decrease of $|\delta \phi|^2$ is more evident, and χ_{AA}^{eff} jumps when the reduced ω_r causes ω_s to switch from positive to negative; $\chi_{AA}^{eff} \downarrow$; $\gamma \downarrow$

$A_z(Z) \uparrow \rightarrow$ total χ_{eff}^e first \uparrow and then \downarrow

11/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

Driven parameters: collisionality ν_{ei}

$\nu_{ei} \uparrow \rightarrow \omega_r \downarrow$, while γ_k increase first and then decrease

$A_z(Z) \uparrow$: for small ν_{ei} , $A_z(Z) \uparrow \rightarrow \gamma_k \uparrow$, and opposite for large ν_{ei}

$\nu_{ei} \uparrow \rightarrow \chi_{AA}^{eff} \downarrow$: because the decrease of $|\delta \phi|^2$ is more evident; χ_{AA}^{eff} increase first and then decrease

12/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

Driven parameters: electron temperature gradient η_e

$\eta_e \uparrow \rightarrow \omega_r \downarrow$, while γ_k increase first and then decrease

When $Z f_c = 0.2$, $A_z(Z) \uparrow \rightarrow \omega_r \downarrow$: for small η_e , $A_z(Z) \uparrow \rightarrow \gamma_k \uparrow$, and opposite for large $\hat{\beta}_e$

$\eta_e \uparrow \rightarrow \chi_{AA}^{eff} \downarrow$: because the decrease of $|\delta \phi|^2$ is more evident; χ_{AA}^{eff} first \uparrow and then \downarrow

13/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

Impurity parameters: $\hat{L}_{ez}(a/L_{ne} = 17.1)$

Besides the type of the impurity, the impurity profile (\hat{L}_{ez}) can also affect the MTM and the transport

$a/L_{nz} \uparrow \rightarrow \hat{L}_{ez} \uparrow \rightarrow K_{eff} \downarrow, \omega_r, \gamma_k \uparrow$

$\hat{L}_{ez} \uparrow \rightarrow$ total $\chi_{eff}^e \uparrow$: because $\gamma_k \uparrow$

14/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

4. Summary

Dispersion relation and transport fluxes at DIII-D strong gradient pedestal:

Main conclusions

- $\nu_{ei} \uparrow$ can make $\omega_r \downarrow$ and γ_k increase first then decrease, $\beta_e \uparrow$ can stabilize MTM
- MTM dominates electron heat transport in the pedestal, as $D_e^{eff} / \chi_{eff}^e \sim 0.1$, and χ_{AA}^{eff} is largest
- $\nu_{ei} \uparrow$ and $\hat{\beta}_e \uparrow$ both enhance the electron heat transport

15/15 EPS Plasma Physics Conference 2026(Shanni Huang) 6.29-7.3, Edinburgh UK

