Modelling of flux pumping in tokamaks

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Introduction

According to the literature [1]-[4], the flux pumping mechanism assumes that the toroidal component of the perturbed non-axisymmetric negative MHD dynamo associated with the central $1/1$ mode helps to counterbalance the central plasma toroidal current increase. The used resulting induction equation is found by subtracting the 2D axisymmetric (no instabilities) component from the more general nonlinear 3D non-axisymmetric equation involving the $n > 0$ instabilities. We intend to basically use the latter instabilities' value directly provided by our model [5], built upon a classical linear approach of a perturbations model from the equilibrium state, in order to further calculate the nonlinear dynamo term.

The perturbed dynamo

In the following, the index 1 is kept from the magnetic flux pumping literature and refers to all the perturbations. The toroidal perturbations dynamo is $(\mathbf{v}_1 \times \mathbf{B}_1)^{\varphi} = (\mathbf{v}_1 \times \mathbf{B}_1)\cdot \mathbf{e}^{\varphi}$, where the contravariant basis vectors $\mathbf{e}^u = \nabla u$ of the natural coordinates $\{u\} = \{r, \theta, \varphi\}$ are used. Similarly, the covariant basis vectors are $\mathbf{e}_u = \partial \mathbf{r}/\partial u$ with $\mathbf{r}$ the position vector. The perturbed $\mathbf{v}_1$ is defined as $\mathbf{v}_1 = \dot{\xi}$ with $\xi = (1/B)\nabla\Phi \times \mathbf{n} + \xi_{||}\mathbf{n}$ by means of the perturbed scalar electric potential $-\dot{\Phi}$ and the displacement parallel to the magnetic field $\xi_{||}$. $\mathbf{n} \equiv \mathbf{B}/B$ is the equilibrium magnetic field unit vector. Thus $(\mathbf{v}_1 \times \mathbf{B}_1)^{\varphi} = (1/\sqrt{g})(v_{1r}B_{1\theta} - v_{1\theta}B_{1r})$ with $\sqrt{g} = \mathbf{e}_r \cdot \mathbf{e}_\theta \times \mathbf{e}_\varphi$ being the coordinates' jacobian. $\mathbf{n} \cong n_\varphi \mathbf{e}^{\varphi}$ is the usual approximation due to the prevalence of the toroidal magnetic field. From (11) and (12) of [5] we get the total magnetic field $B = (R_0 B_{z0}/\sqrt{g_{\varphi\varphi}})\sqrt{1 + g_{\theta\theta}/(q^2 g_{\varphi\varphi})}$, where R_0 is the major radius, B_{z0} the equilibrium magnetic field at the magnetic axis and q the local safety factor. According to our model the perturbed magnetic field is introduced using a non-resistive approach as $\mathbf{B}_1 = \nabla \times (\xi \times \mathbf{B})$. The perturbed model we use basically solves the ideal plasma perturbed equations whose boundary conditions consist of perturbed jump equations across inertial resistive plasma layers disconnecting two ideal regions. The general solution of the entire system of equations Φ^{mn} [5] associated with a

*See the author list of C.F. Maggi et al 2024 Nucl. Fusion 64 112012 https://doi.org/10.1088/1741-4326/ad3e16
†See the author list of N. Vianello et al "Results from the last DD and DT JET campaigns in the framework of the EUROfusion Tokamak Exploitation Work Package Activity" 2026 Nucl. Fusion 66 116010 https://doi.org/10.1088/1741-4326/ae71ec

mode of perturbation is chosen to be located at the ideal/resistive frontier. Under the assumption of a negligible skin current, the current perturbation being concentrated inside the layer, the ideal parametrization of $\mathbf{B}_1$ is approximated as being valid. However, the general solution obviously fully considers the resistivity effect of the inertial plasma layers, due to the perturbed jump equations across it that are used into calculus. This approach has been proven to work by simply checking the good match between the derived modes amplitude, frequency and location and its similar experimental quantities. Therefore, unlike the usual models solving the perturbed equations inside the resistive plasma region, our approach allows the ideal-like, resistivity free parametrization of the above perturbed magnetic field to be considered. It is just a formal approach, the plasma inertial layer resistivity being there, englobed within the ξ quantity. Within the large aspect ratio approximation, by keeping into account the metric coefficients (A.2)-(A.6) of [5] and dropping the higher powered inverse aspect ratio terms, we get the following flux surface averaged dynamo term

$$\langle(\mathbf{v}_1\times\mathbf{B}_1)^{\varphi}\rangle=\sum_{m,n}\langle(\mathbf{v}_1\times\mathbf{B}_1)^{\varphi}_{mn}\rangle-\frac{a\varepsilon}{rR_0^4q^2B_{z0}} \tag{1}$$

$$\times\sum_{m,n}n(m-nq)\sum_{\substack{j=-3\\ j\neq 0}}^{3}\left[(-1)^{|j|!}r^2\Lambda'_{|j|}\dot{\bar{\Phi}}'^{m+j,n*}-\frac{|j|}{2j}\left(\frac{r^2}{a\varepsilon R_0^2}\delta_{|j|1}-\Xi_{|j|}\right)(m+j)\dot{\bar{\Phi}}^{m+j,n*}\right]\Phi^{mn}$$

with $\langle f\rangle=\int_0^{2\pi}\int_0^{2\pi}d\theta d\varphi\sqrt{g}\,f/\left(\int_0^{2\pi}\int_0^{2\pi}d\theta d\varphi\sqrt{g}\right)$. $*$ means complex conjugation, ! factorial and δ_{ij} is the Kronecker symbol. a is the minor radius and $\varepsilon=a/R_0$. The Fourier component Φ^{mn} is derived via the plasma perturbed momentum and adiabatic equations considering the pressure gradient that ultimately drives the flux pumping. $\dot{\bar{\Phi}}^{mn}=\dot{\Phi}^{mn}-\mathrm{i}n\Omega_{\varphi}\Phi^{mn}$ considering the toroidal plasma velocity $\Omega_{\varphi}=\dot{\varphi}$ as the prevalent rotational velocity of the plasma. Φ'^{mn} is not the radial derivative of the Φ^{mn}, but the similarly local radial derivative of the general Φ and is the complementary solution of the local perturbed system of equations (see [5]). There is no measurable quantity proportionally linked with Φ'^{mn}. $\{\Lambda_j\}_{j=1,2,3}=\{\Delta(r),E(r),T(r)\}$ where $\{(r^2/R_0)\Lambda_j\}_{j=1,2,3}$ are the local magnetic surface Shafranov shift, ellipticity and triangularity, respectively. $\Xi_j=(j^2-1)\Lambda_j+r\Lambda'_j+r^2\Lambda''_j$. The uncoupled, single mode term of (1) is

$$\langle(\mathbf{v}_1\times\mathbf{B}_1)^{\varphi}_{mn}\rangle\cong\frac{1}{r^2R_0^2qB_{z0}} \tag{2}$$

$$\times\left\{m\left[ms\Phi^{mn}-r(m-nq)\Phi'^{mn}\right]\dot{\bar{\Phi}}^{mn*}-r(m-nq)\left(m+\frac{nr^2}{qR_0^2}\right)\Phi^{mn}\dot{\bar{\Phi}}'^{mn*}\right\}$$

For the present flux pumping analysis, this is to be further used for the $1/1$ mode only, due to the significantly lower amplitudes of the neighboring modes. Although the last term of (2) is of an expendable $O(\varepsilon^2)$ order, it still has been kept due to the low central safety factor at denominator that could make the term comparable with the others.

Results

The toroidal induction equation for the flux surface averaged quantities written in terms of varying perturbations from an equilibrium state is $\langle\eta J^{\varphi}\rangle=(\mathbf{v}_0\times\mathbf{B}_0)^{\varphi}+\langle(\mathbf{v}_1\times\mathbf{B}_1)^{\varphi}\rangle$, with v_0

and B_0 the equilibrium velocity and magnetic field, respectively. The aim is to compare the experimentally measured left hand side of the equation with the modelled right hand side. The toroidal current density is provided by the equilibrium reconstruction data whereas the Spitzer resistivity make use of the HRTS and ZEFF data, both quantities being fitted at the $1/1$ mode magnetic surface location, if we restrict ourselves to this perturbation only. They are already flux surface averaged, time dependent radially located quantities. In order to get rid of the equlibrium quantities we simply vary the equation in time. Therefore, we get at every moment of time

$$\Delta(\eta J_\varphi)^{exp} \equiv (\eta J_\varphi)^{exp} - (\eta J_\varphi)^{exp}\big|_{t_0} = E_{dyn} \tag{3}$$

where $E_{dyn} = R_0\langle(\mathbf{v}_1 \times \mathbf{B}_1)^\varphi\rangle$. R_0 relates the contravariant to the measurable quantity ($|\mathbf{e}_\varphi| = \sqrt{g_{\varphi\varphi}} \cong R_0$). The initial time t_0 is chosen so that the perturbed dynamo initial contribution is almost zero, that basically qualifies t_0 as the zero amplitude $1/1$ mode onset time. By us-

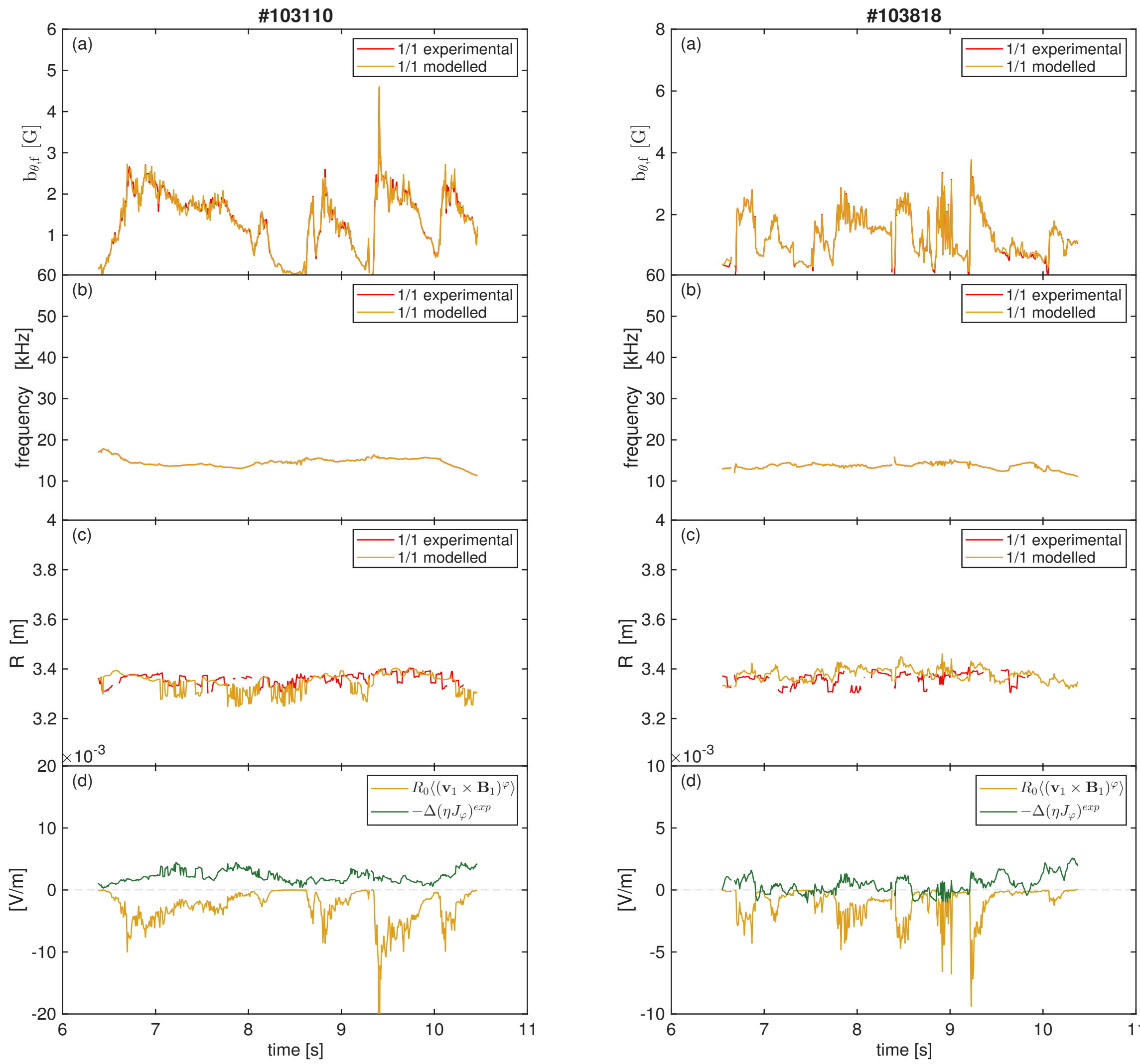

Figure 1: *JET 103110 (left) and 103818 (right) shots experimental vs modelled* $1/1$ *mode (a) amplitude, (b) frequency and (c) location. The* $1/1$ *mode toroidal dynamo voltage loop vs. the time variation of the experimental toroidal resistive electric field is depicted in (d).*

ing the calculated mode amplitude $b_{\theta f}^{mn} = (m/q-n)(mr^m/R_0 r_f^{m+1})|\Phi^{mn}|$, frequency $f^{mn} =$

$\mathrm{Im}[(\partial\Phi^{mn}/\partial t)/\Phi^{mn}]$ and location derived as shown in [6], the precise retrieval of the experimental corresponding quantities is just a measure of the model accuracy in this case, as seen in figure 1(a-c) for the case of JET shots no. 103110 and 103818 belonging to a dedicated flux pumping experimental campaign. r_f is the location of the JET coils where amplitude is measured. There is a clear, good match with the experimental quantities provided by the JET data analysis code [7]. Based on this, our calculated $1/1$ toroidal component of the dynamo voltage loop is depicted in figure 1(d) against the negative time variation of the experimental toroidal central electric field. The shapes of the curves balance each other reasonably good enough, although their magnitudes could more or less slightly differ due, in our opinion, to the imperfect mode location match and to the many used approaches. In figure 2 the magnetic axis safety factor q_0 is inversely derived via the well-known Wesson formula experiencing plasma shaping corrections, starting from the modelled $1/1$ mode safety factor.

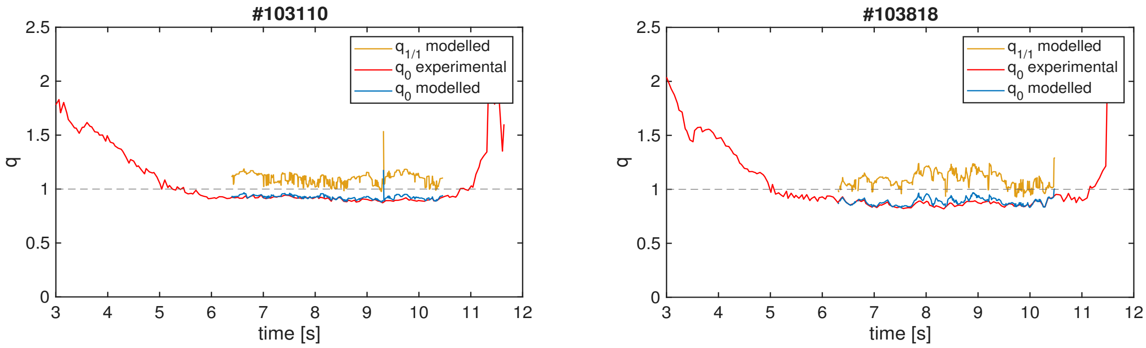

Figure 2: *JET 103110 (left) and 103818 (right) shots modelled* $1/1$ *and experimental vs. modelled magnetic axis safety factors.*

Conclusions

No radial variation of the perturbed terms of induction equation is derived therefore, no radial flattening/broadening of the central plasma current density is proved or assumed. A temporal voltage balance to describe the mechanism rather than a spatial (radial) balance has been used instead, being limited by the localized perturbations model we use. From figure 1(d) it is clear that the perturbed $1/1$ mode dynamo loop voltage prevents the central toroidal electric current increase in time. If not the resistivity, then the current density accumulation is prevented, that is actually what flux pumping theory is basically claiming.

This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200-EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

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