Amplification and saturation of hot plasma waves driven by runaway electrons in tokamaks

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Experimental and theoretical studies are extensively pursued, concerning runaway electron (RE) formation in tokamak plasmas and the evolution in time of their distribution function. These studies and their importance for fusion reactors are discussed in [1]. In recent years, the detection of radiofrequency (RF) waves, emitted in the presence of RE in tokamaks [2-5], has opened the search for a new kind of diagnostics. The identification of RF waves driven by RE is indeed a promising method to infer the characteristics of the RE distribution. The wave instabilities can be driven by two RE free energy sources: the anisotropy of the RE distribution, with velocities almost parallel to the magnetic field, and its local peaking in the momentum space, often predicted. Two main interactions occur: the Cherenkov and the anomalous Doppler effects. Linear stability analysis of cold plasma waves driven by RE considering any resonance order, with arbitrary RE distribution functions and plasma equilibrium configurations has been discussed in [6]. There, the effective amplification of the unstable waves has been then evaluated by means of a ray-tracing code and based on a statistical analysis of several rays. In [7], the linear stability analysis was performed considering hot plasma waves, by the code REDHPW. Here we propose a linear stability analysis of IBW driven unstable by RE. The model is implemented by the numerical code FAREDW (Finite Amplitude Runaway Electron Driven Waves). For any point P chosen on a magnetic surface where RE are present, a preliminary evaluation of the frequencies of waves driven unstable by RE is performed. The initial wave energy density spectrum in wavenumber space $dW_{\boldsymbol{k}}/d^3\boldsymbol{k}$ is evaluated based on the fluctuation-dissipation theorem.

This gives

$$\frac{dW_{\boldsymbol{k}}}{d^3\boldsymbol{k}} = \frac{k_B T_i}{(2\pi)^3} \frac{\boldsymbol{E}_{\boldsymbol{k}}^* \bar{\bar{\varepsilon}}^A \boldsymbol{E}_{\boldsymbol{k}}}{\boldsymbol{E}_{\boldsymbol{k}}^* \left[\frac{\partial}{\partial \omega} \omega \bar{\bar{\varepsilon}}^H\right]_{\omega=\omega_{\boldsymbol{k}}} \boldsymbol{E}_{\boldsymbol{k}}} \quad (2)$$

Here $\bar{\bar{\varepsilon}}^A$ and $\bar{\bar{\varepsilon}}^H$ are, respectively, the anti-Hermitian and the Hermitian part of the Maxwellian plasma dielectric tensor. The collisional effects are included adopting the substitution $\omega \rightarrow \omega + i\nu_\alpha$ in the conductivity tensor of the plasma species α [7]. Since the ion Bernstein mode is primarily sustained and damped by the ion response, the effective fluctuation temperature entering the fluctuation–dissipation relation is taken to be T_i. This approximation follows from the dominance of the ion contribution to the anti-Hermitian dielectric tensor. The ray-tracing technique is then used to calculate the wave packet propagation and amplification from the initial point P, up to the last point where the eikonal approximation is still valid. As a reference example, we consider the TCV Deuterium plasma discharge SN 86969, where RE are present and RF line emissions as clusters with almost fixed frequency gap were observed. In particular, at the observation time $t = 0.7$ s, have been detected line emissions with frequency gap corresponding to the cyclotron frequency of Deuterium for the magnetic field on axis, namely 1.43 T. The main plasma parameters are reported in Table I. We have limited the present analysis in the frequency range 80-100 MHz, where line emissions at about 87 MHz and 98 MHz have been observed. Temperatures and the density linearly decreasing with ψ are assumed with T_e = 1.14 keV, T_i = 0.5 keV, $n_e = 1.36\ 10^{19}$ m^{-3} at $\psi = 0$ and $T_e = T_i$ = 30 eV, $n_e = 5\ 10^{11}$ m^{-3} at $\psi = 1.0$. Since no evidence of significant interaction via anomalous Doppler has been observed around the observation time, it is assumed that the Cherenkov wave-beam interaction is dominant and that the main free energy source is provided by a local peak of the RE distribution function in the momentum space. A skew normal energy distribution and almost gaussian angular distribution is adopted, as already used and described in [7]. In the (E, μ) space this distribution function is given by

$$f(E, \mu) = \frac{n_b}{C\sigma_E} \varphi\left(\frac{E - E_p}{\sigma_E}\right) \Phi\left(\alpha \frac{E - E_p}{\sigma_E}\right) \frac{\exp(\sigma_\theta \mu)}{2\sigma_\theta \sinh \sigma_\theta}. \quad (3)$$

Here C is a normalization constant, such that the integral of f in the (E, μ) space is the beam density n_b, $E_p = 7.5$ MeV, $\sigma_E = 4.0$ MeV, $\sigma_\theta = 20, \alpha = 1$. A linear profile of the RE density vs ψ is assumed decreasing from a maximum on axis to zero at $\psi = 0.25$.

In Figure 1, the perpendicular components of the refractive index and the IBW group velocity are plotted vs. the wave frequency, as obtained from the wave dispersion equation for the parallel refractive index $N_{\parallel} = 1.0022$. In Figure 2, we plot the RE distribution function and the IBW growth rates at 0.2% RE concentration, the ion cyclotron and the collisional damping rates expected near the magnetic axis for $N_{\parallel} = 1.0022$. In Figure 3, the net IBW growth rates and the normalized wave energy density in wavenumber space due to thermal fluctuations, as expected near the magnetic axis, are plotted vs. the frequency.

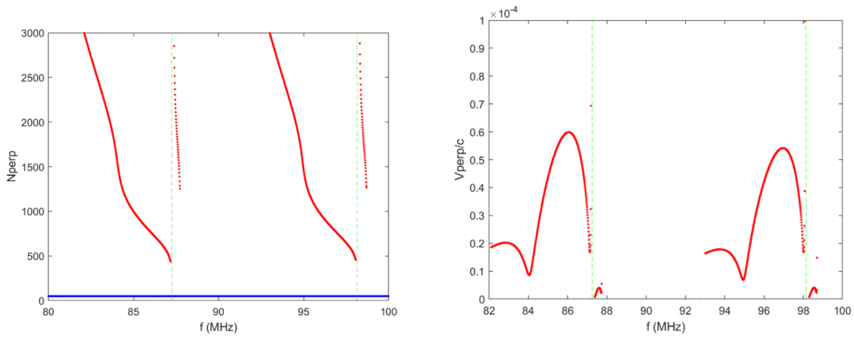

Figure 1. The perpendicular refractive index (left), with the IBW branches in red and the WW branch in blue, and the perpendicular group velocity of the IBW branches (right) are plotted vs. the wave frequency. The vertical green dashed lines indicate the 8th *and 9*th *ion cyclotron harmonics resonances. Here* $N_{\parallel} = 1.0022$.

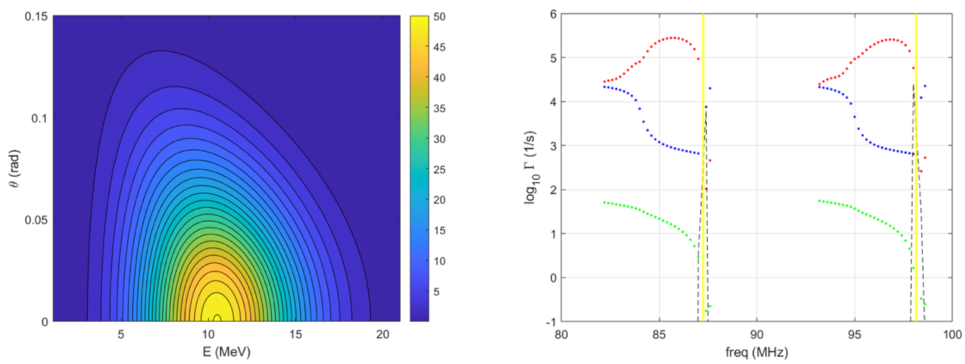

Figure 2. Contour plot (left) of the function $g(E, \theta) = f(E, \cos\theta)$, *where* f *is the distribution function of RE in the* (E, μ) *space, Eq.* (3). *Growth rates (red), collisional damping rates due to electrons (green) and ions (blue), as well as the ion cyclotron damping rates (dashed line) of IBW are plotted vs. the wave frequency (right). The yellow vertical lines indicate the 8*th *and 9*th *ion cyclotron resonance.*

The present analysis suggests that IBW produced by thermal fluctuations with frequency near the observed values of 87 MHz and 98 MHz can be amplified by RE around the magnetic axis. Ray-tracing of IBW packet in the range 86.5 – 87.5 MHz has been performed in cylindrical coordinates (Fig. 4, left), with $N_{\parallel} = 1.0015 - 1.0030$ and $\eta = 0^o - 1.0^o$. η is the angle between the vector $\boldsymbol{B} \times \nabla\psi$ and the perpendicular wave number $\boldsymbol{k}_{\perp}$.

As a result, single pass energy density amplification by IBW-RE interaction due to Cherenkov resonance is found, with saturation level of 60-65 dB above the thermal noise reached in a time interval of the order of 10 μs (Fig. 4, right) for 1% RE concentration on axis, carrying 82.5 kA current. The rays crossing ion-cyclotron harmonics resonant layers are completely damped [8], possibly producing ion heating effects.

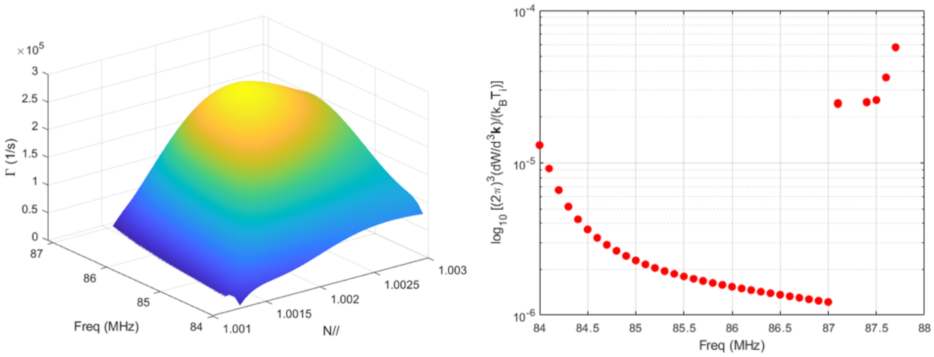

Figure 3. Plots of the net growth rate of IBW driven by RE vs the parallel refractive index and the wave frequency (left) and normalized energy density per unit volume in the wavenumber space vs the wave frequency (right).

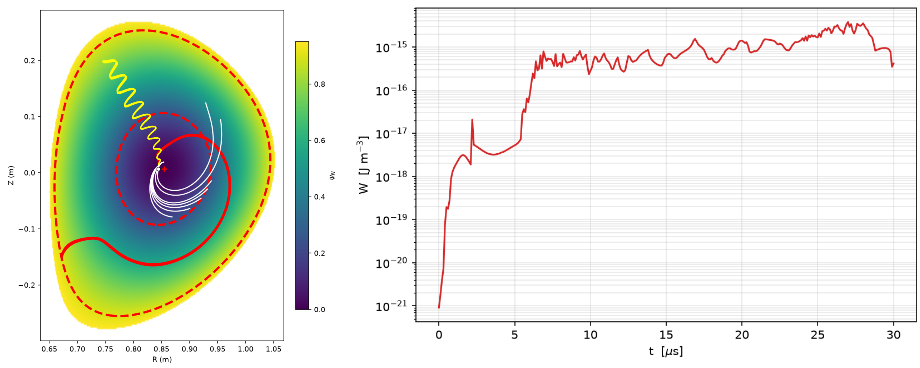

Figure 4. Ray tracing (left) of IBW packets with $N_{\parallel} = 1.0022$*,* η *from* 0^o *to* 1.0^o*, for* 86.5 MHz *(red)* 87.0 MHz *(yellow) and* 87.5 MHz *(white). Energy density vs time (right) of an IBW packet with* $N_{\parallel} = 1.0015 - 1.0030$*,* $f = 86.5 - 87.5$ MHz*,* $\eta = 0^o - 1.0^o$.

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