Amplification and saturation of hot plasma waves driven by runaway electrons in tokamaks

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Experimental and theoretical studies are extensively pursued, concerning runaway electron (RE) formation in tokamak plasmas and the evolution in time of their distribution function. These studies and their importance for fusion reactors are discussed in [1]. In recent years, the detection of radiofrequency (RF) waves, emitted in the presence of RE in tokamaks [2-5], has opened the search for a new kind of diagnostics. The identification of RF waves driven by RE is indeed a promising method to infer the characteristics of the RE distribution. The wave instabilities can be driven by two RE free energy sources: the anisotropy of the RE distribution, with velocities $\mathbf{v}$ almost parallel to the magnetic field $\boldsymbol{B}$, and its local peaking in the momentum space, often predicted. Two main interactions occur: the Cherenkov and the anomalous Doppler effects. Linear stability analysis of cold plasma waves driven by RE considering any resonance order, with arbitrary RE distribution functions and plasma equilibrium configurations has been discussed in [6]. There, the effective amplification of the unstable waves has been then evaluated by means of a ray-tracing code and based on a statistical analysis of several rays. In [7], the linear stability analysis was performed considering hot plasma waves, by the code REDHPW. Here we propose a linear stability analysis of IBW driven unstable by RE. The model is implemented by the numerical code FAREDW (Finite Amplitude Runaway Electron Driven Waves). For any point P chosen on a magnetic surface where RE are present, a preliminary evaluation of the frequencies of waves driven unstable by RE is performed. The initial wave energy density spectrum in wavenumber space $dW_{\boldsymbol{k}}/d^3\boldsymbol{k}$ is evaluated from the thermal noise level, based on the fluctuation-dissipation theorem.

This gives

$$\frac{dW_{\boldsymbol{k}}}{d^3\boldsymbol{k}} = \frac{1}{2}\frac{k_B T_i}{(2\pi)^3} \quad (1)$$

Since the ion Bernstein mode is primarily sustained and damped by the ion response, the effective fluctuation temperature entering the fluctuation–dissipation relation is taken to be T_i. This approximation follows from the dominance of the ion contribution to the anti-Hermitian dielectric tensor. The ray-tracing technique is then used to calculate the wave packet propagation and amplification from the initial point P, up to the last point where the eikonal approximation is still valid. As a reference example, we consider the TCV Deuterium plasma discharge SN 86969, where RE are present and RF line emissions as clusters with almost fixed frequency gap were observed. In particular, at the observation time $t = 0.7$ s, have been detected line emissions with frequency gap corresponding to the cyclotron frequency of Deuterium for the magnetic field on axis, namely 1.43 T. We have limited the present analysis in the frequency range 80-100 MHz, where line emissions at about 87 MHz and 98 MHz have been observed. The temperatures and the density are assumed linear decreasing functions of ψ, with T_e = 1.14 keV, T_i = 0.5 keV, n_e = 1.36 10^{19} m^{-3} at $\psi = 0$, $T_e = T_i$ = 30 eV, n_e = 5 10^{11} m^{-3} at $\psi = 1.0$., with 7% fully ionized ^{12}C. Since no evidence of significant interaction via anomalous Doppler has been observed around the observation time, it is assumed that the Cherenkov wave-beam interaction is dominant and that the main free energy source is provided by a local peak of the RE distribution function in the momentum space. A skew normal energy distribution and almost gaussian angular distribution is adopted, as already used and described in [7]. In the (E, μ) space, where $\mu = \mathbf{v}\cdot\boldsymbol{B}/(|\mathbf{v}||\boldsymbol{B}|)$, this distribution function is given by

$$f(E,\mu) = \frac{n_b}{C\sigma_E}\varphi\left(\frac{E-E_p}{\sigma_E}\right)\Phi\left(\alpha\frac{E-E_p}{\sigma_E}\right)\frac{\exp(\sigma_\theta\mu)}{2\sigma_\theta \sinh\sigma_\theta}. \quad (2)$$

Here C is a normalization constant, such that the integral of f in the (E,μ) space is the beam density n_b, $E_p = 7.5$ MeV, $\sigma_E = 4.0$ MeV, $\sigma_\theta = 20, \alpha = 1$. A linear profile of the RE density vs ψ is assumed, decreasing from a maximum on axis to zero at $\psi = 0.25$. In Figure 1, the perpendicular components of the refractive index and the IBW group velocity are plotted vs. the wave frequency, as obtained from the wave dispersion equation.

The parallel refractive index $N_{\parallel} = 1.0022$ is chosen. In Figure 2, we plot the RE distribution function and the IBW growth rates at 0.2% RE concentration, the ion cyclotron and the collisional damping rates expected near the magnetic axis, again for $N_{\parallel} = 1.0022$. In Figure 3, the net IBW growth rates are plotted vs. the frequency and the parallel refractive index.

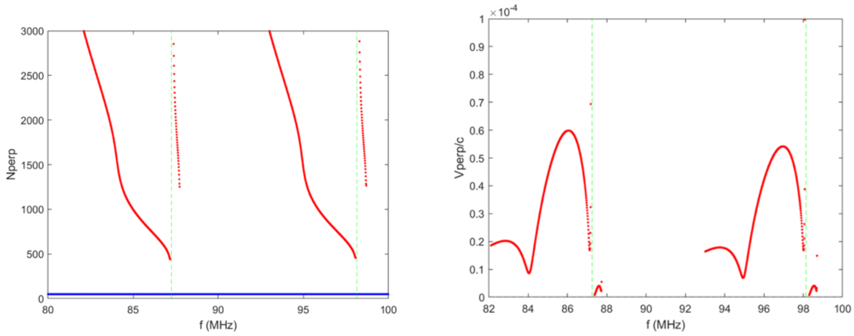

Figure 1. The perpendicular refractive index (left), with the IBW branches in red and the WW branch in blue, and the perpendicular group velocity of the IBW branches (right) are plotted vs. the wave frequency. The vertical green dashed lines indicate the 8th and 9th ion cyclotron harmonics resonances. Here $N_{\parallel} = 1.0022$.

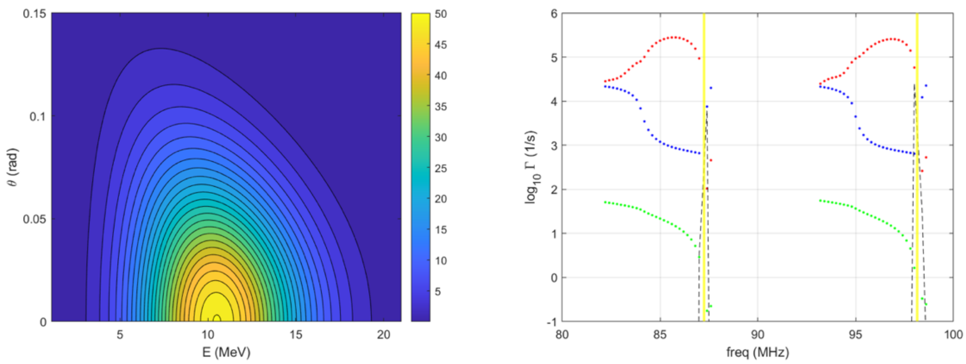

Figure 2. Contour plot (left) of the function $g(E, \theta) = f(E, \cos\theta)$*, where* f *is the distribution function of RE in the* (E, μ) *space, Eq.* (3)*. Growth rates (red), collisional damping rates due to electrons (green) and ions (blue), as well as the ion cyclotron damping rates (dashed line) of IBW are plotted vs. the wave frequency (right). The yellow vertical lines indicate the 8th and 9th ion cyclotron resonance.*

The analysis performed suggests that IBW produced by thermal fluctuations with frequency near the observed values of 87 MHz and 98 MHz can be amplified by RE around the magnetic axis. Ray-tracing of IBW packet in the range 86.8 – 87.0 MHz has been performed in cylindrical coordinates (Fig. 4, left), with $N_{\parallel} = 1.0015 - 1.0040$ and $\eta = 175^{o} - 179$. η is the angle between the vector $\boldsymbol{B} \times \nabla\psi$ and the perpendicular wave number $\boldsymbol{k}_{\perp}$. As a result, single pass energy density amplification by IBW-RE interaction is found, with saturation at least 25 dB above the thermal noise level, reached in a time interval of the order of 30 μs (Fig. 4, right) for 1% RE concentration on axis, hence 82.5 kA RE current, i.e. about 38.5 % of the total current.

The rays crossing ion-cyclotron harmonics resonant layers are completely damped [8], possibly producing ion heating effects. Further work is required to explore $(N_{\parallel}, \eta)$ space at higher resolution and compute the frequency spectra considering all spatial origins of the IBW packets.

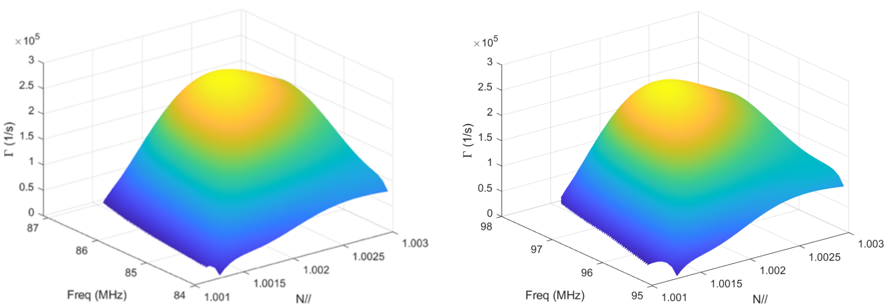

Figure 3. Plots of the net growth rate of IBW driven by RE vs the parallel refractive index and the wave frequency in the range 84-88 MHz *(left) and* 95-98 MHz *(right) for plasma, magnetic field and beam parameters on axis.*

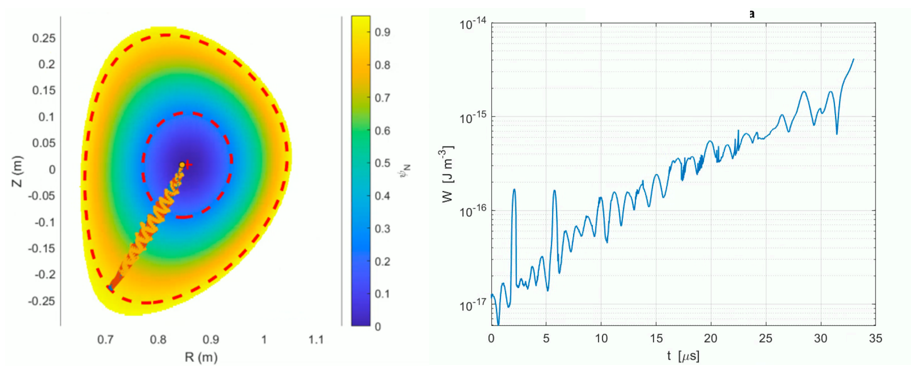

Figure 4. Ray-tracing of IBW packets (left) with $N_{\parallel} = 1.0018 - 1.0040$, *with steps* 10^{-4}, *η from* 175^{o} *to* 179^{o} *in steps of* 0.1^{o}, *for 86.8 MHz, 86.9 MHz and 87.0 MHz. Dashed, red lines correspond to* $\psi_N = 0.25$ *and* 0.9. + *sign mark indicates the position of the magnetic axis. Amplification of the energy density of IBW packet (right).*

The initial seed should be evaluated taking into account turbulence effects. Quasilinear theory should be used to evaluate the time evolution of RE distribution function consistently with IBW amplification. Nonlinear mode excitation via inverse Landau damping should be considered.

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