Drift wave turbulence in "full-f" and "full-k"

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Introduction

Models and simulations of drift wave edge turbulence in magnetically confined plasmas generally make use of various simplifications. A common approximation is the "delta-f" form of equations, where only small turbulent perturbations (relative to the average "background" quantities) are evolved. As in the edge and scrape-off layer of tokamaks and stellarators fluctuation amplitudes can be large in the order of the background, over recent years several "full-f" (or "total-f") gyrokinetic, gyrofluid and fluid codes have been developed around the world. Full-f gyrokinetic and gyrofluid models are for consistency usually considering only long-wavelength approximated polarization. Going beyond this limit, Padé-based arbitrary wavelength polarization closures for full-f models have recently been formulated by Held et al. [1].

Here, we report on our implementation and testing of solvers with 2nd order accurate finite-Larmor radius (FLR) polarization in "full-f full-k" gyrofluid codes [2, 3] for resistive drift wave turbulence, and on ongoing further developments of thermal and multi-ion models. Numerical solvers were developed by means of the 2d gyrofluid Hasegawa-Wakatani code "TIFF" [2], and applied in the 3d electromagnetic isothermal toroidal flux-tube code "T3FF" [3].

2D full-f full-k gyrofluid code TIFF

The 2D code TIFF [2] is a test platform for the development of full-f full-k gyrofluid polarization solvers. Implemented test cases include a gyrofluid Hasegawa-Wakatani turbulence model, or interchange blob propagation. The full-f polarization equation with 2nd order (“full-k”) FLR effects [1] is given (in normalized form) as a generalized Poisson equation:

$$\sum_s \nabla \cdot \sqrt{\Gamma_{0s}}\, \varepsilon_s(x,y) \sqrt{\Gamma_{0s}} \nabla_\perp \phi(x,y) = \sigma(x,y) \tag{1}$$

with $\sigma = -\sum_s Z_s \Gamma_{1s} N_s$ and $\varepsilon_s = m_s N_s / B^2$, where $N_s(x,y)$ and the gyrocenter densities of electrons and ions with species index $s \in (e,i)$, and the Padé based gyro-operators are $\Gamma_{0s} = 1/[1 + (\rho_s k_\perp)^2]$ and $\Gamma_{1s} = 1/[1 + (\rho_s k_\perp)^2/2]$. For details see ref. [2]: there we had tested and compared several polarization solvers for a single ion species and in the approximation $\varepsilon_e \sim m_e \equiv 0$. For this we have implemented a standard 4th order pre-conditioned conjugate gradient scheme

(PCG) and standard 2nd order successive-over-relaxation (SOR) scheme, and compared the accuracy and performance with a newly devised Teague-like 4th order "dynamically corrected Fourier" (DCF) method. The DCF scheme for a generalized variable-coefficient Poisson problem $\nabla \cdot \hat{\varepsilon} \nabla \hat{\phi} = \hat{\sigma}$, obtains in each time step of a dynamical simulation the new electric potential $\hat{\phi}(x,y)$ by a Fourier method corrected via the previous steps $\hat{\phi}^{old}$:

$$\hat{\phi}(x,y) = \nabla^{-2} \left\{ \nabla \cdot \frac{1}{\hat{\varepsilon}_i} \nabla \left(\nabla^{-2} \hat{\sigma} \right) + \left[\frac{1}{\hat{\varepsilon}_i}, \nabla^{-2} \left[\hat{\phi}^{old}, \hat{\varepsilon}_i \right] \right] \right\} \tag{2}$$

where $[a,b]$ is the Poisson bracket, and $\nabla^{-2} = -\mathscr{F}^{-1} \mathbf{k}^{-2} \mathscr{F}$ is evaluated by FFTs. The scheme is an efficient and (for small time steps) sufficiently accurate (non-iterative) solver with predictable run times [2]. We note that it could also be used recursively as a (stand-alone) iterative solver.

3D electromagnetic full-f full-k gyrofluid codes: T3FF / TEFF / T6FF

The codes T3FF [3] and TEFF evolve the isothermal electromagnetic full-f gyrofluid equations [4, 5] in a field-aligned flux tube with simple circular $s-\alpha$ torus geometry or a tabulated metric. The same "full-f full-k" polarization eq. (1) is used with a choice between the PCG or DCF solvers. The full-f Ampere's law (neglecting FLR effects on the normalized vector potential $A_{||}$) is solved with a PCG or an SOR Helmholtz solver on each z plane:

$$\nabla_{\perp}^2 A_{||} - \hat{\beta} \sum_s \frac{Z_s}{\hat{\mu}_s} N_s(x,y,z)\, A_{||} = -\sum_s \frac{Z_s}{\hat{\mu}_s} N_s(x,y,z)\, P_s(x,y,z) \tag{3}$$

The set of evolution equations for the gyrocenter densities N_s and the generalized momenta $P_s = \hat{\mu}_s u_s + \hat{\beta} A_{||}$ is solved with a three-step Adams-Bashforth method, and a 4th order Arakawa scheme for the (advective and magnetic flutter) nonlinear Poisson brackets. Parallelization is with OpenMP (in the published version ref. [3]) or hybrid OpenMP/MPI. In ref. [3] the code verification and benchmark with respect to standard edge turbulence test cases have been reported. For full grid convergence a resolution of at least 2 nodes per drift scale ρ_0 in the (x,y) directions and $n_z = 32$ nodes along the flux tube (local magnetic field) direction is required.

The code is presently being further extended. The version "TEFF" includes an edge/SOL coupling with (limiter) sheath boundary conditions in z direction on the open field line region. An example for the resulting turbulent electron density fields $N_e(x,y)$ on four of the parallel (z) planes are shown in fig. 1 (a): the closed field line edge region comprises a $64\rho_0 \times 256\rho_0$ domain and the open (sheath b.c.) region extends another $32\rho_0$ in the radial (x) direction.

A "T6FF" code version which includes the 6-moment set of full-f thermal gyrofluid equations is presently under development and undergoing first testing.

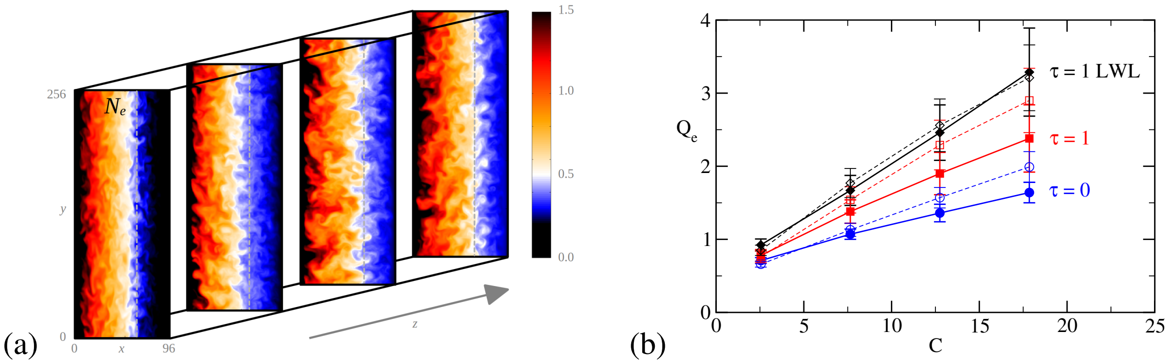

Figure 1: *(a) Isothermal full-f full-k electromagnetic gyrofluid code "TEFF": turbulent density $N_e(x,y)$ in several field-aligned flux-tube sections (z) of a (s-α) tokamak edge / SOL region. (b) Full-k (red, $\tau = T_i/T_e = 1$) density transport Q_e deviates significantly from long-wavelength approximated results (black). Dashed lines connect results in the respective delta-f limits.*

Multi-species full-f full-k polarization solver

In the long-wavelength limit of $\Gamma_{0s} \to 1$, the full-f polarization equation $A\phi = \sigma$ takes the form of a generalized, variable-coefficient Poisson equation when $A = \sum_s \nabla \cdot \sqrt{\Gamma_{0s}}\varepsilon_s\sqrt{\Gamma_{0s}}\nabla_\perp$ $\to \nabla \cdot [\sum_s \varepsilon_s]\nabla_\perp$. This can be solved with standard methods (e.g. PCG, DCF or SOR) also for multiple (ion) species s, as shown in ref. [6]. The full-f gyrofluid approach allows a consistent treatment of secondary ions (e.g. D-T mixture) or localized non-trace impurities. The standard solvers need to be enhanced when treating also Γ_{0s} effects for all (massive / ion) species s. We are presently implementing and testing several iterative approaches (also in context of [7]).

Here, we show an implementation as a matrix-free spectral solver by adapting the PCG scheme, and defining the spectral operator $A = \mathscr{F}^{-1}[\sum_i i\mathbf{k}\sqrt{\Gamma_{0s}}\mathscr{F}\varepsilon_i\mathscr{F}^{-1}[i\mathbf{k}\sqrt{\Gamma_{0s}}\mathscr{F}\phi]]$. A diagonal preconditioner $P(k_x,k_y) = |k|^2\sum_i \bar{\varepsilon}_i\Gamma_{0s}(k_x,k_y)$ with average $\bar{\varepsilon}_i$ is used. The initial residual is defined by $r = \phi - A\phi^{old}$ with the result $\phi^{old}$from the previous time step. The precondition is set as $p \equiv z = \mathscr{F}^{-1}P^{-1}\mathscr{F}r$. The iteration proceeds with standard PCG updates:

(i) $\phi \leftarrow \phi + \alpha p, \quad r \leftarrow r - \alpha A p \quad$ with $\alpha = (r,z)/(p,Ap)$;

(ii) $p \leftarrow z + \beta p, \quad$ with $\beta = (r,z)/(r,z)_{old} \quad$ and precondition $z = \mathscr{F}^{-1}P^{-1}\mathscr{F}r$;

and iterated until the residual r is below a specified tolerance. For two (massive, non-trace) ion species the procedure requires 8 FFTs for the initialization and per iteration step. For a resolution of 2 grid points per ρ_0 around $3-10$ iterations to an error $e \sim 10^{-4}$ are needed. The run time is approximately $2-3$ times that of a single-ion solver. Scenarios with e.g. D-T mixtures of similar edge density profiles require significantly fewer iterations than with localized impurity clouds. The model and solvers now allow for simulations of self-consistent non-trace full-f impurity advection with 2nd order FLR effects (beyond the long-wavelength approximation).

2nd order FLR ("full-k") vs. long-wavelength approximation

The delta-f limit of the (single-ion) polarization eq. (1) can be written as $\Gamma_0 \nabla_\perp^2 \phi = \sigma$. When the extra FLR effects through the Γ_0 operator are neglected in the long-wavelength limit (LWL), results of (resolution converged) simulations significantly differ from those of a "full-k" model. Exemplary average transport levels Q_e from T3FF drift-Alfvén turbulence simulations ($n_{x,y}/L_{x,y} = 4, n_z = 32$) are shown in fig. 1 (b) for various collisional parameters C in full-f and delta-f cases: LWL models overestimate transport for all present parameters. Further common simplifications (mostly in gyrokinetic core ITG studies) take $\varepsilon(x,y)$ in eq. (1) to be approximately constant or a flux-surface averaged profile $\varepsilon(x)$: we find that (even for edge gradient cases) such reductions impose less additional deviations than the primary LWL approximation.

Outlook

Version T6FF is being extended as a 6-moment model, and our solvers are being optimized and tested with GPU support [7]. We also like to cross reference the full-f gyrofluid work by Wiesenberger and Held on the full torus geometry code project FELTOR (see e.g. refs. [5, 8, 9]).

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