

Introduction

- Quasilinear models combine linear modelling of turbulent instability growth rates, with a saturation rule predicting maximum instability amplitude, to calculate turbulent fluxes for vastly lower computational cost than gyrokinetic simulations
- Current best-in-class gyrofluid based quasilinear model (TGLF) struggles in the high β conditions expected in next generation spherical fusion reactor designs
- Changes to TGLF's numerical settings improve the prediction of the onset of β' stabilisation, and modelling of the effects of $E \times B$ shear, although a significant underestimation of the suppression remains

TGLF

- The TGLF linear solver is a Gyrofluid Eigenvalue solver, where separate moments are taken for the trapped and passing fraction of particles. The resulting gyrofluid equations are solved as an eigenvalue problem using Hermite basis functions

$$\Psi_n = e^{-\hat{v}^2/2} H_n, \quad \int_{-\infty}^{\infty} d\hat{v} \Psi_m \Psi_n = \delta_{n,m}, \quad \hat{\theta} = \theta \theta_w.$$

- Initially TGLF fits the instability eigenfunction with **NBASIS_MIN** (default: 2) Hermite basis functions, and the width of the Gaussian measure is adjusted between **WIDTH_MIN** and **WIDTH_MAX**, considering the electrostatic only (ES) or all fields (EM) to get the maximum growth rate. Hermite Basis functions up to **NBASIS_MAX** are added to better resolve the eigenfunction

TGLF Numerical Settings

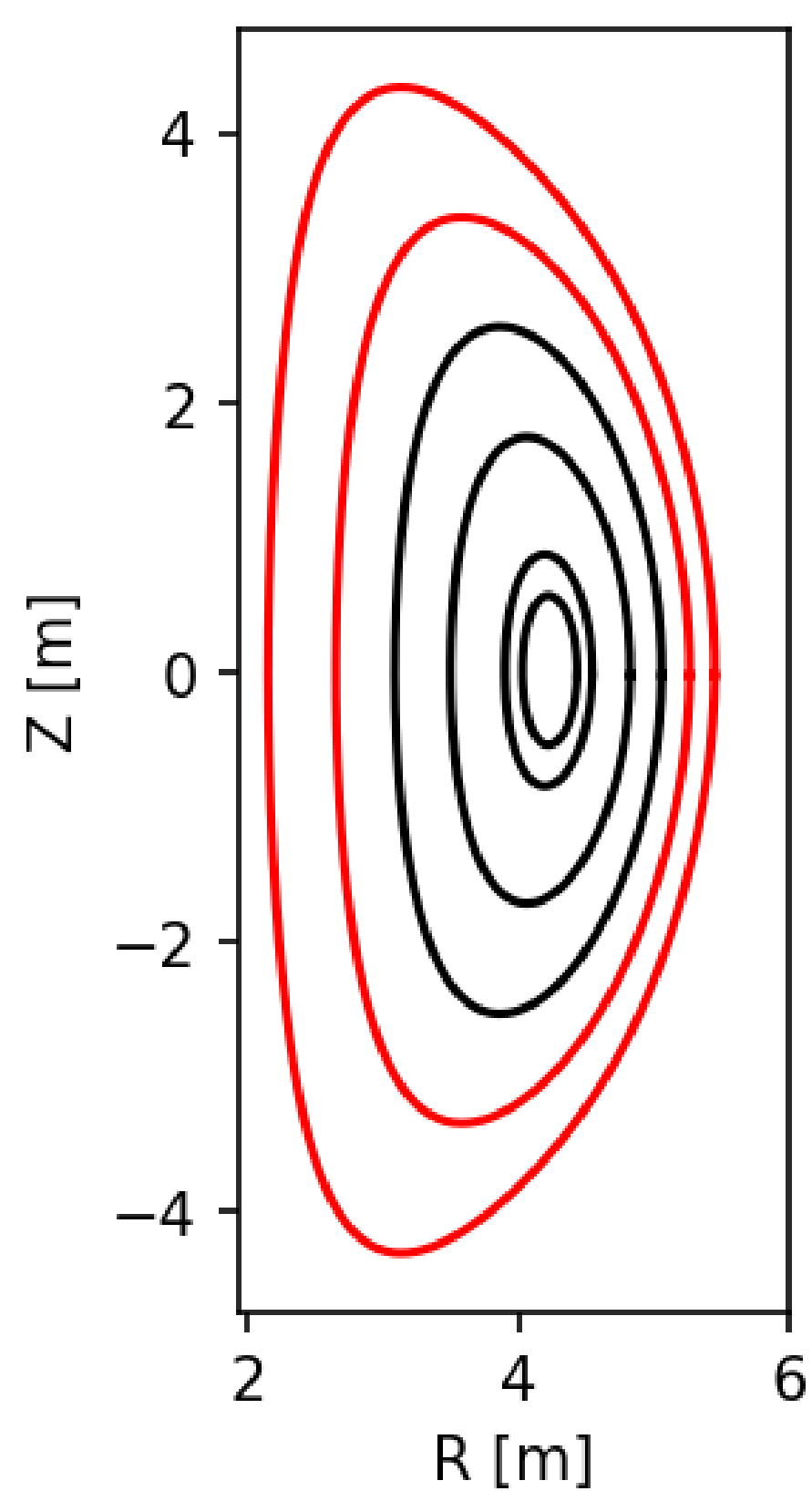


Figure 1. Plot of the magnetic flux surfaces for the T3D relaxed STEP-EC-HD case, with the specific radial cases used in R3 ($\rho = 0.655$) and R4 ($\rho = 0.818$) shown in red

- Previous work found that increasing **NBASIS_MAX** improved agreement between TGLF and the gyrokinetic code GS2 for 4 radial cases of the T3D relaxed STEP scenario STEP-EC-HD [2] [4] (TGLF V1). However, a high filtering setting was necessary to match growth rates for the R4 radial case
- Disabling width search and reducing **NBASIS_MAX** removes the need for aggressive filtering and improves agreement (TGLF V3)

Numerical Params	Default	V1	V3
NBASIS_MAX	4	10	4
WIDTH	1.65	1.9	1.3
NXGRID	16	32	32
THETA_TRAPPED	0.7	0.57	0.7
FIND_WIDTH	ES	ES	F

Table 1. Comparison of TGLF numerical parameters.

Linear Comparison

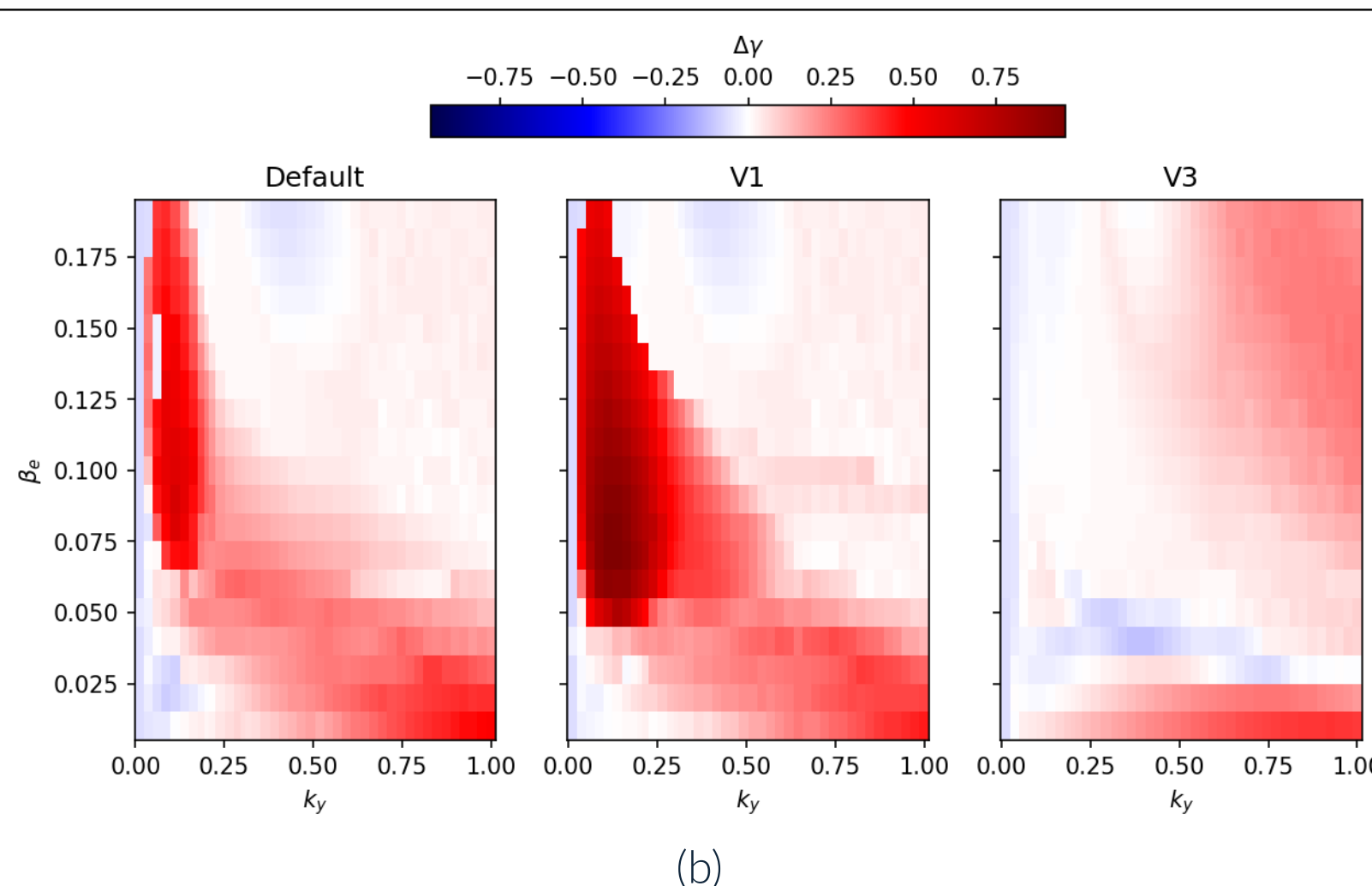
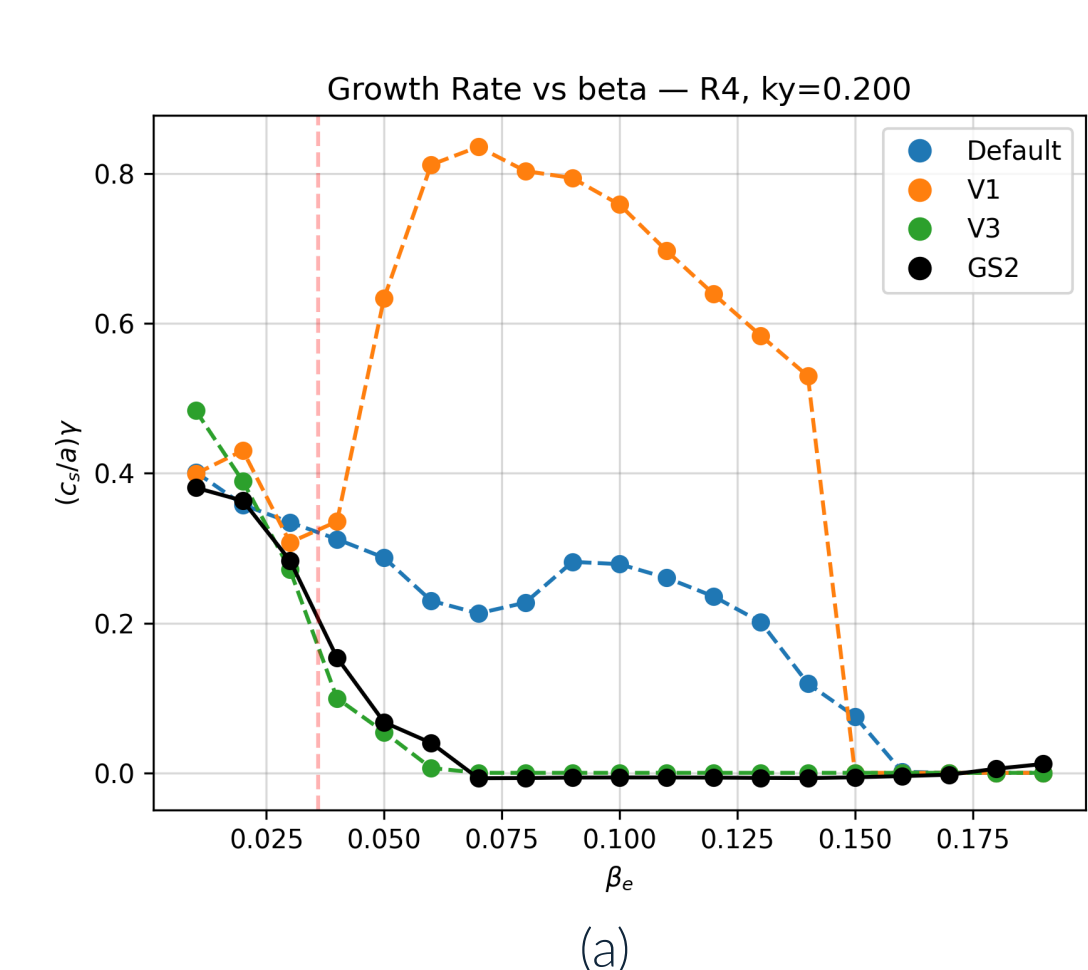


Figure 2. (a): Growth rate against β_e (β' varied consistently) plot for the R4 radial case from GS2 linear simulations against TGLF with default and improved numerical settings and GS2 GP against β_e at $k_y \rho_s = 0.2$. The red vertical line is the reference value for β . (b): Colour Heatmap of absolute error in growth rates relative to GS2 for each TGLF version.

- V3 improves the agreement particular at low k_y due to the removal of spurious low width KBMs fig. 2b and clearly demonstrates Beta prime stabilisation fig. 2a
- V3 more closely matches the effects of varying the safety factor for the STEP-EC-HD case fig. 3b, although a demonstration of increasing magnetic shear disestablishing KBMs is absent fig. 3a. Varying q is expected to move the beta prime stability boundaries, moving turbulence towards the first stability region at low q and second stability region at high q
- When compared across a dataset based on the STEP-EC-HD geometry fig. 4, V3 has dramatically lower root mean square error in it's prediction of the growth rate

Case	Field	Version		
		Default	V1	V3
MAST 48567	γ	0.585	0.739	0.390
	ω	1.620	2.140	1.740
R3	γ	0.059	0.102	0.052
	ω	0.335	0.574	0.307
R4	γ	0.291	0.330	0.352
	ω	0.972	0.915	1.180
STEP-EC-HD	γ	0.041	0.072	0.054
	ω	0.373	0.783	0.412

Table 2. Mean absolute error in growth rate and mode frequency for each case against GS2.

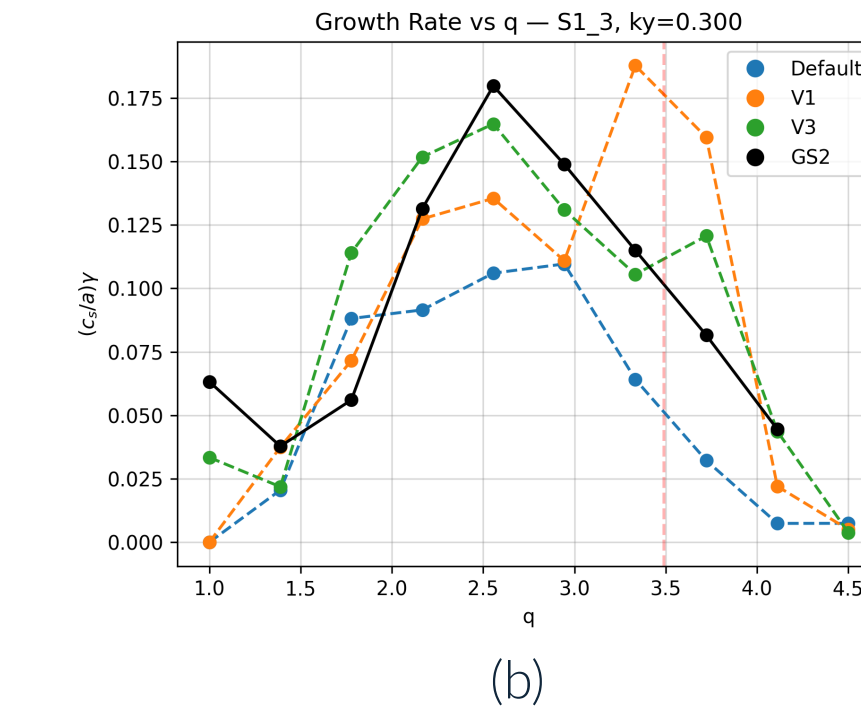
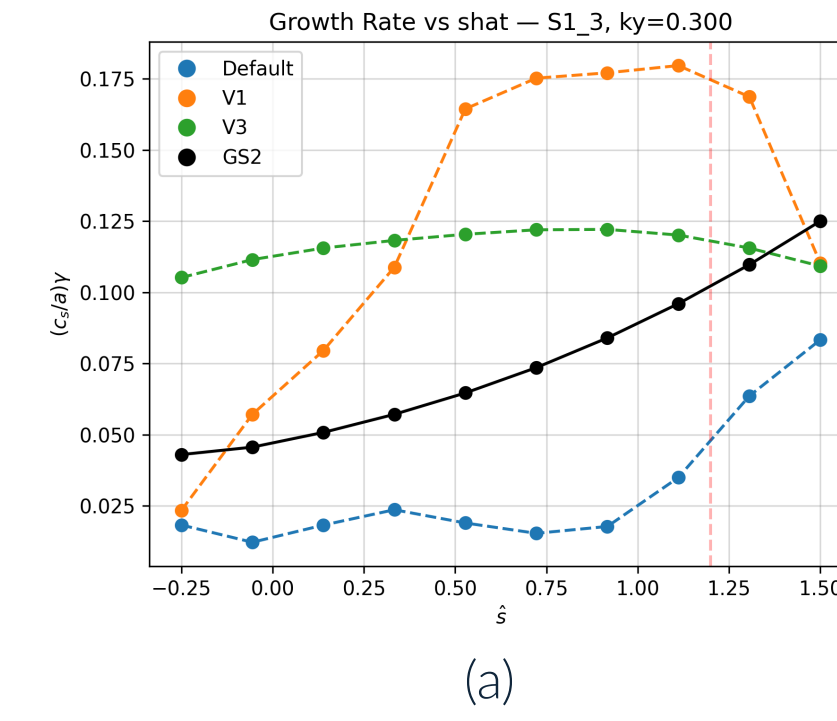


Figure 3. Growth rate plots for the STEP-EC-HD case against magnetic shear (a) and safety factor (b). The red vertical line in each case is the reference value of magnetic shear and safety factor respectively

	MAST 48567	R3	R4	STEP-EC-HD
shift	-0.368	-0.406	-0.505	-0.399
ρ	0.718	0.655	0.818	0.637
κ	1.760	2.560	2.610	2.560
κ'	0.088	0.015	0.194	0.015
δ	0.168	0.294	0.402	0.283
δ'	0.200	0.432	0.922	0.292
q	-2.39	3.580	5.310	3.490
\hat{s}	1.630	1.160	1.970	1.200
β	0.036	0.095	0.053	0.010
β'	0	-0.262	-0.758	-0.056
a/L_T (D)	3.240	0.790	4.240	1.820
a/L_T (E)	3.260	1.590	2.210	1.580
a/L_n (E)	3.060	0.097	4.270	1.030
a/L_n (D)	3.060	0.097	4.270	1.030
ν_e	0.351	0.043	0.057	0.023

Table 3. Shaping parameters and gradient in the 4 cases considered

TGLF vs GS2 Growth Rates (STEP-EC-HD Geometry)

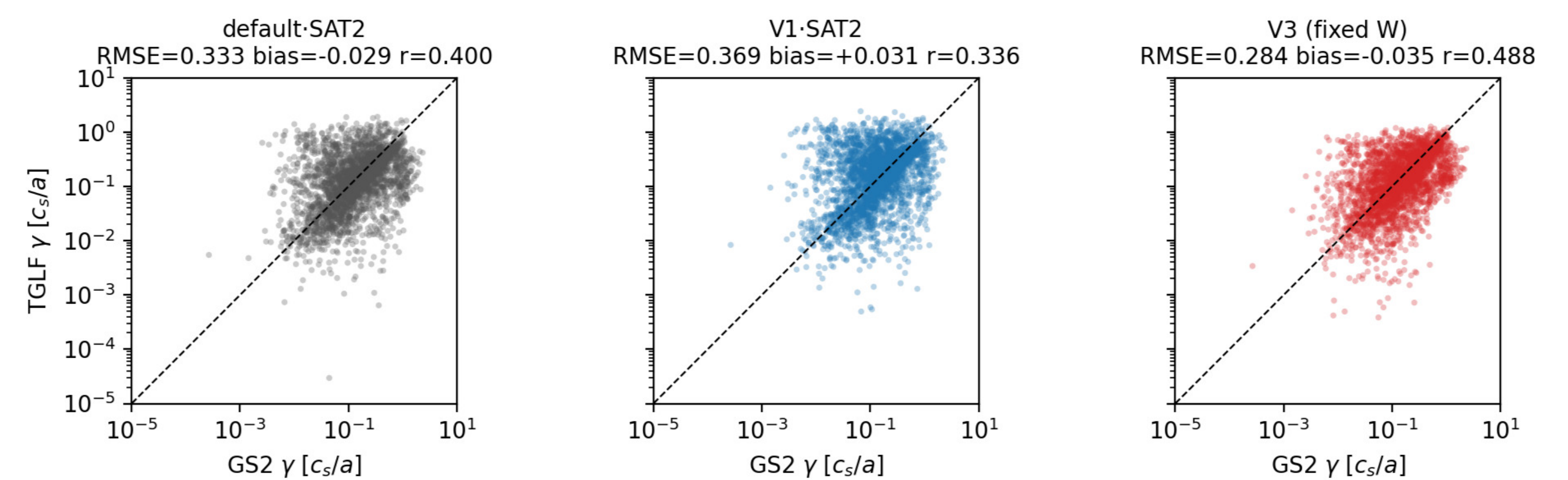


Figure 4. Comparison of TGLF growth rate against GS2 for 3000 data points generated from the STEP-EC-HD geometry. 8 parameters were varied including: q , \hat{s} , β , ν_e , electron temperature and density gradients and ion temperature gradient. Points were sampled at ky values ranging from 0.05 and 1

Heat Flux Comparison

- TGLF couples its gyrofluid linear solver with 3 options for a saturation rule, [7] [5] [1]. The saturation rules acts to estimate the flux amplitude from the linear growth rate spectrum, however all saturation rules need a peak in the growth rate over k_y spectrum, which is not consistently produced. $E \times B$ shear is modelled as a damping in the growth rate spectrum before the saturation rule is applied.
- TGLF V1 performs especially well in the STEP-EC-HD case fig. 5a, however requiring Sat 1. V3 performs poorly in the STEP-EC-HD case predicting full turbulence suppression and only marginally better in the MAST Case fig. 5b

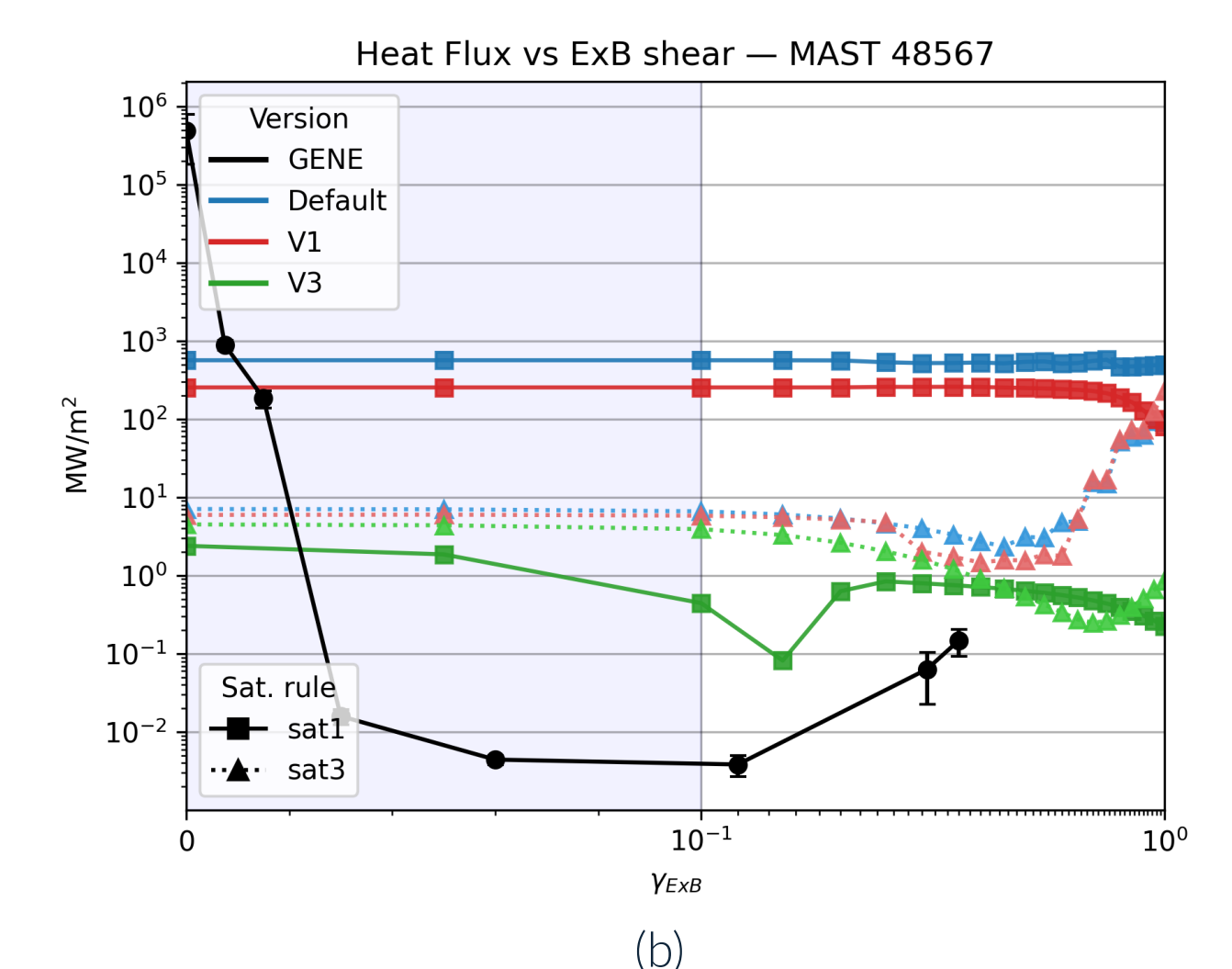
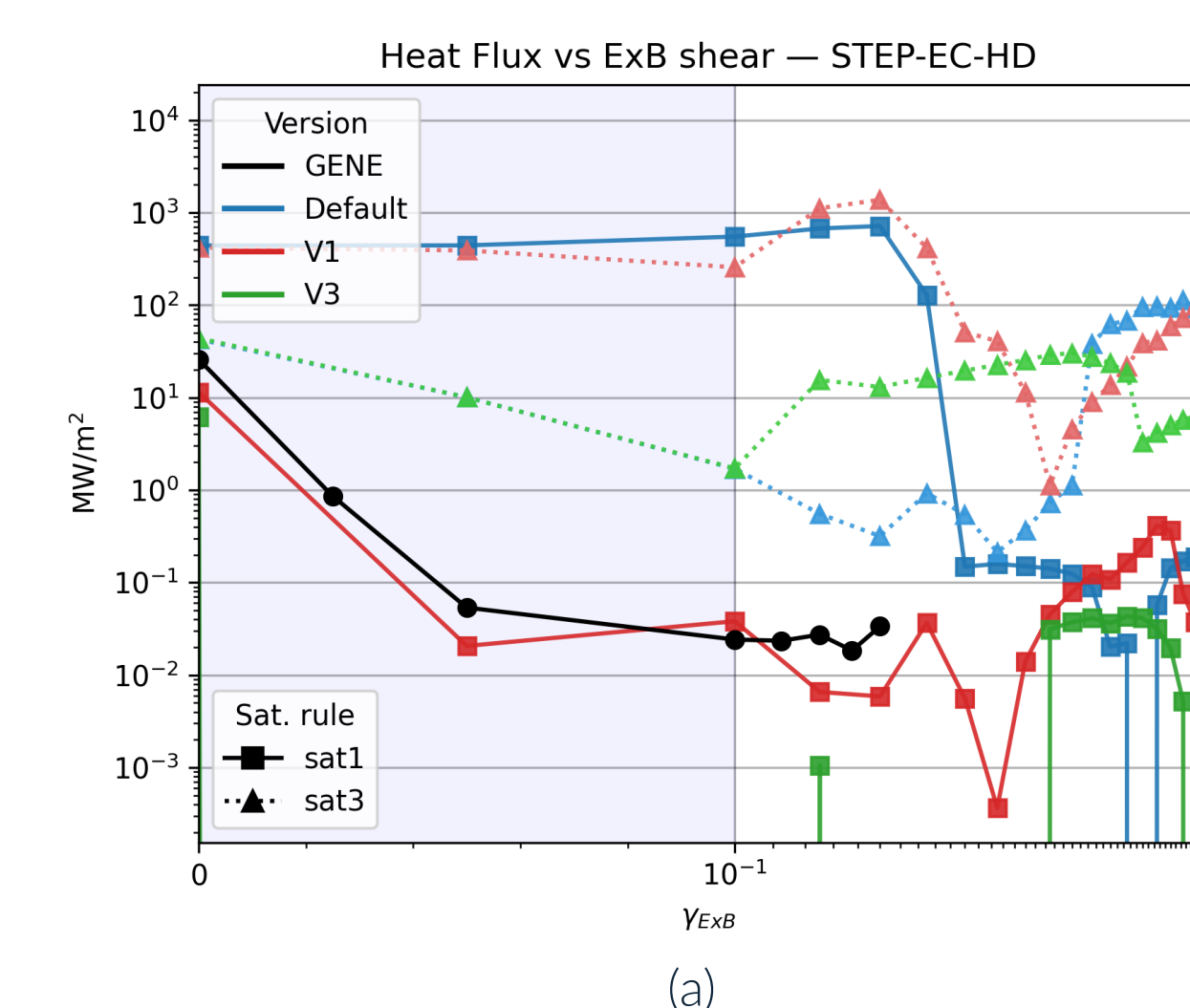


Figure 5. Plot of heat flux against $E \times B$ shear rates for the STEP case STEP-EC-HD (a) and MAST shot 58567 (b) respectively. TGLF with different numerical settings (differentiated by colours) and saturation rules (differentiated by symbols) are compared to a GENE non-linear gyrokinetic simulation. The region for $E \times B$ (0-0.1) is plotted linearly with the remainder plotted logarithmically

Future Work

- Alternatives to quasilinear models include machine learning surrogates. A Gaussian regression model based on the linear gyrokinetic code GS2 has been developed [3], and preliminary results show a $\sim 2\times$ reduction in RMSE against TGLF, within the models training space
- The spectral shift model [6] models $E \times B$ shear as a shift and suppression in the k_x spectrum. Non-linear gyrokinetic analysis of high β cases suggests $E \times B$ shear causes a greater suppression in the k_x spectrum at lower shift than expected. Updating the TGLF saturation rules may allow a far better prediction of the effects of $E \times B$ shear

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