Axisymmetric ($n = 0$) high-frequency MHD oscillations at the pedestal of JET plasmas

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Long-lived axisymmetric ($n = 0$) quasi-coherent high frequency oscillations (HFOs, $50 - 500$ kHz) have been reported close to the L-H transition, during both low and high confinement modes, at JET [1] and AUG [2] tokamaks. The L-H transition is characterised by the appearance of an $n = 0$ $m = 1$ (up-down) low frequency coherent and multi-harmonic magnetic oscillation (LFO, with fundamental harmonic of order $0.5 - 2$ kHz) that modulates the pedestal gradient. It is known as the M-mode at JET [3] and the I-phase at AUG [2]. In RF heated plasmas with slow power ramps, the frequency of the M-mode is proportional to the edge poloidal Alfvén frequency [3]. HFOs are modulated by these pedestal oscillations and we found that they carry an Alfvénic signature as well. Both LFOs and HFOs are measured in the poloidal magnetic signal $\dot{B}_\theta$ and can be visualised in the processed spectrogram of $\dot{B}_\theta$ displaying the toroidal mode number, as illustrated in figure 1, where blue corresponds to $n = 0$. Despite some proposed models [4], the nature of HFOs is still elusive.

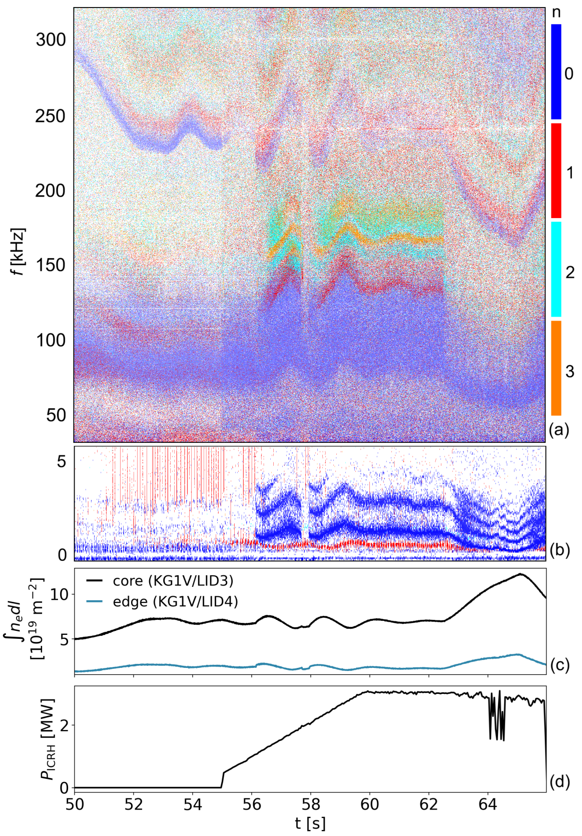

Figure 1: *Toroidal mode number spectrogram in the (a) high and (b) low frequency range, blue being* $n = 0$*; (c) core and edge line integrated electron density; (d) ICRH heating power for JET discharge #80951.*

Although initially reported at the L-H transition, we found that HFOs are not restricted to such scenarios and appear in a variety of plasma compositions (H, D, D-He^3, D-T) with a wide range of plasma parameters and under different heating schemes, namely pure Ohmic, neutral beam injection (NBI) and ion cyclotron resonance heating (ICRH). While the intensity of the bands is not affected by the presence of NBI, whose signal remains as strong as during the

Ohmic heating phase, ICRH appears to suppress the highest-frequency band. Besides, the latter is considerably attenuated during the H-mode. We further found that $n = 0$ HFOs are generally accompanied by $n = 1$ bands that share an identical modulation, hinting at their Alfvénic nature.

Here, we show that axisymmetric HFOs can be understood in light of MHD theory as $n = 0$ Global Alfvén Eigenmodes (GAEs).

Besides JET and AUG, modes of Alfvénic nature with similar properties have been reported on TFTR [5], MAST [7], and COMPASS [6], and identified as GAEs in the latter two.

Global Alfvén Eigenmodes and axisymmetric shear-Alfvén Continuum

Ideal MHD theory states that only a class of discrete shear Alfvén Eigenmodes (AEs) can exist for $n =$ 0: the Global Alfvén Eigenmodes (GAEs). These are discrete modes of plasma oscillation that arise due to and near a local extremum of the shear-Alfvén continuum (SAC) [8]. In the cylindrical limit, the $n = 0$ SAC reduces to $\omega^2 \approx v_A^2 m^2/(R^2 q^2)$, with v_A the Alfvén speed, q the safety factor, m the Fourier harmonic number and R the distance to the torus axis. Thus, the sharp decrease of plasma density along with the increase of q at the plasma edge give rise to a local minimum close to the plasma boundary.

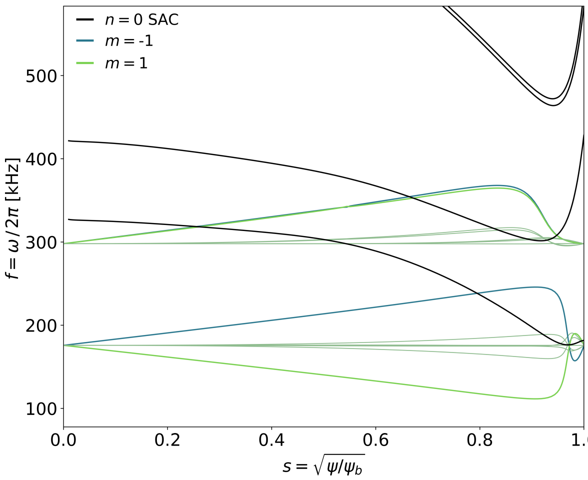

Figure 2: *Axisymmetric shear-Alfvén continuum computed by* `CSMISH` *for discharge #82221 at* $t =$ 48.07 *s. The radial component of the perturbed velocity,* v_s*, of the GAEs found by* `MISHKA` *under the SAC first and second minimum is plotted as well, emphasising the dominant* $m = \pm 1$ *harmonics.*

Figure 2 shows the axisymmetric SAC computed by `CSMISH` [9] along with the two GAEs found by the ideal MHD code `MISHKA` [10] below the two lowest branches of the continuum, whose frequencies differ little from the respective SAC minimum (4 kHz for the upper mode and 1 kHz for the lower one). This small difference motivated the derivation of an analytic expression for the two lowest branches of the axisymmetric SAC which can be used as a proxy for the frequency of GAEs and, ultimately, of HFOs.

Analytic expression for the lowest branches of the coupled SAC

The two lowest branches of the continuum illustrated in figure 2 come, in a first approach, from the coupling of the Fourier harmonics $m = \pm 1$ and consequent opening of a continuous gap that extends from the core to the plasma edge, separating the branches that would otherwise coincide (in the cylindrical limit, ω^2 depends exclusively on m^2).

Using a local magnetic equilibrium model with up-down asymmetric cross section [11], we derived a first order expression for the lowest branches of the coupled SAC to disclose their dependence on geometrical parameters.

Linearising the set of ideal MHD equations under the small-perturbation formulation of the plasma displacement $\xi e^{i\omega t}$ and dropping all compressible terms ($\nabla \cdot \xi = 0$), the shear-Alfvén continuous spectrum is formed by the set of values ω^2 that yields non-trivial solutions for [12]

$$\left[\frac{\omega^2 \rho |\nabla\Psi|^2}{B^2} + B \cdot \nabla \left(\frac{|\nabla\Psi|^2}{B^2} B \cdot \nabla\right)\right] \xi^A = 0, \tag{1}$$

where $\xi^A = \xi \cdot B \times \nabla\Psi / |\nabla\Psi|^2$ corresponds to the shear-Alfvén (or binormal) component of the plasma displacement, B is the equilibrium magnetic field, and Ψ is the poloidal-field flux function, parametrised as

$$\Psi(r,\theta) = \Psi_b S_0 r^2 \Big[\Theta_0(\theta) + \varepsilon r \Theta_1(\theta)\Big]. \tag{2}$$

In the latter expression, Ψ_b is the poloidal-field flux at the boundary, S_0 is a geometrical parameter inversely proportional to the cylindrical q, $\varepsilon = a/R_0$ is the small parameter expansion (being a and R_0 the minor and major radius, respectively), $\Theta_0(\theta)$ and $\Theta_1(\theta)$ are angular functions that introduce geometric coefficients related to the shafranov shift, the plasma elongation (κ), etc. In particular, $\Theta_0(\theta) = 1 + \kappa_s \cos 2\theta + \kappa_a \sin 2\theta$, where κ_s relates with κ as $\kappa \approx \sqrt{\frac{1+\kappa_s}{1-\kappa_s}}$.

Expanding ξ^A in Fourier series as $\xi^A = \sum_m \xi^A_m(\Psi) e^{im\theta}$ (where axisymmetry, $n = 0$, is implicit) and discarding all contributions coming from Fourier harmonics other than $m = \pm 1$ such that $\xi^A = \xi^A_{-1} e^{-i\theta} + \xi^A_1 e^{i\theta}$, the resulting expression for the two lowest branches of the coupled continuum is

$$\tilde{\omega}^2_{\pm} = \frac{2 \pm \kappa_s}{\tilde{\rho} q^2 (2 \mp \kappa_s)}, \tag{3}$$

where $\tilde{\omega} = \omega R_0 / v_A(0)$, $\tilde{\rho} = \rho / \rho(0)$ and second order terms in $\kappa_s \sim \varepsilon$ were dropped.

Thus, the factor responsible for the gap opening in a first-order approximation is the plasma ellipticity. In the limit $\kappa_s^2 \ll 1$, the size of the continuum frequency gap is $(\omega^2_+ - \omega^2_-) \propto \kappa_s$, in agreement with previous numerical results [13].

Numerical results in agreement with experimental observations

To demonstrate that HFOs are indeed axisymmetric GAEs, we plotted the experimental frequencies (f_{HFO}) against the correspondent $n = 0$ SAC minimum (f_{SAC}), which differ little from the expected numerical GAEs frequency. We opted for this pragmatic approach to save a significant amount of computational effort and to address situations where both `MISHKA` and ideal `CASTOR` [14] failed to calculate modes below the first SAC minimum. This limitation can, however, be overcome by introducing non-ideal effects (namely the resistivity) in the set of ideal MHD equations. In particular, resistive `CASTOR` computes global modes with complex frequency, whose real part is bound to the ideal MHD SAC minima.

Regardless, the results obtained (figure 3) reveal a remarkable agreement and show that ideal MHD can accurately predict the frequencies of axisymmetric HFOs, even though non-ideal contributions may be relevant to the numerical calculation of the modes. The analysis was carried out for a comprehensive set of discharges covering a wide set of plasma compositions (H, D, D-He^3, D-T) and plasma parameters ($I_p = 1.2 - 3.0$ MA, $B = 1.8 - 3.85$ T, $n_e(0) = 2.4 - 6.8$ m^{-3}) that display up to three HFO bands in the spectrogram of $\dot{B}_\theta$.

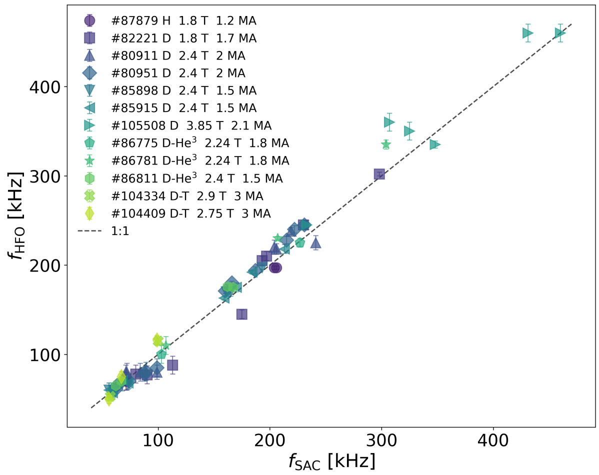

Figure 3: *Measured $n = 0$ HFOs frequency versus the frequency of the correspondent $n = 0$ shear-Alfvén continuum minimum computed by* `CSMISH`.

Summary and future prospects

Axisymmetric HFOs observed at JET and AUG are not restricted to L-H transition phases and appear as a general phenomena in a variety of plasma scenarios. We demonstrated that their frequency can be accurately predicted by ideal MHD theory.

The ease with which these modes are destabilised in a broad range of scenarios suggests they are likely to arise in future reactors, such as ITER. Due to their low amplitude, HFOs are not expected to affect the machine's operation. Nevertheless, they may serve a practical purpose that could prove extremely useful: given their tight link to the lowest branches of the SAC minima, whose analytic expression we derived up to first order terms in κ_s and ε, these modes constitute convenient MHD markers for accurate equilibrium reconstruction near the plasma edge, something that is often lacking.

Acknowledgments

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