Extending the GRAY beam-tracer beyond the standard Electron Cyclotron Heating modelling

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Introduction

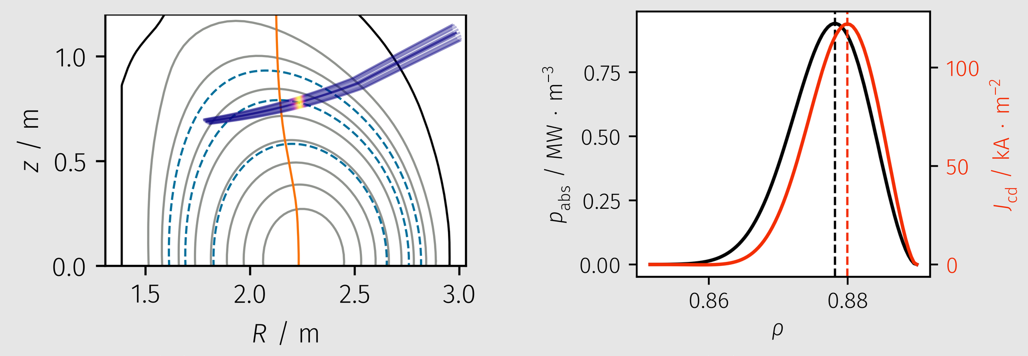
Electron Cyclotron (EC) waves are playing an increasingly larger role in operating fusion devices: from heating (ECRH) and current drive (ECCD), to realtime instability control and plasma breakdown assistance. Finally, the interaction with runaway electrons (RE) is being explored to study their dynamics.

GRAY [1] is a well-established beam-tracing code and a valuable tool for modeling EC waves. Its use has so far been mostly limited to predictive modelling of standard ECRH scenarios, but efforts have been made to extend its applicability across several fronts.

GRAY

GRAY is a **beam-tracer** for EC waves in a tokamak [1].

- **Inputs**: MHD equilibrium, kinetic profiles, beam parameters
- **Outputs**: beam trajectory, absorbed **power**, driven **current**



(a) trajectory (b) profiles

Example GRAY outputs for an NTM control setup

Theoretical framework:

- raytracing: quasi optics, **cold plasma** theory
- heating: relativistic **hot plasma** theory
- current drive: **adjoint method**, neoclassical response

Source code: https://maxwell.eurofusion.eu/git/istp/gray

GRAY in a nutshell

Complex eikonal ansatz: $\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\left[\mathbf{e}(\mathbf{r})e^{iS_R(\mathbf{r})-S_I(\mathbf{r})-i\omega t}\right]$

Gaussian beam → **N rays** obeying raytracing eqs.:

$$\frac{d\mathbf{r}}{ds} = -\frac{\partial D}{\partial \mathbf{k}} \Big/ \left|\frac{\partial D}{\partial \mathbf{k}}\right| \qquad \frac{dS_I}{ds} = 0$$

$$\frac{d\mathbf{k}}{ds} = +\frac{\partial D}{\partial \mathbf{r}} \Big/ \left|\frac{\partial D}{\partial \mathbf{r}}\right| \qquad \mathbf{D}(\mathbf{r},\mathbf{k},\omega)\mathbf{E} = 0$$

where: $\mathbf{k}=\nabla S_R$ wavevector, s arclength, $D = \det \mathbf{D}(\mathbf{r},\mathbf{k},\omega)$ quasi optical dispersion relation.

1. Integrate the raytracing eqs.
2. At each step, for each ray:
   - solve **hot dispersion relation** → absorption coeff. α → power $dP(s) = \alpha P ds$
   - compute current drive efficiency R → current $dI(s) = R dP(s)$
3. Compute $J_{cd} = dI/dA$ and $p_{abs} = dP/dV$ profiles

Hot dispersion relation

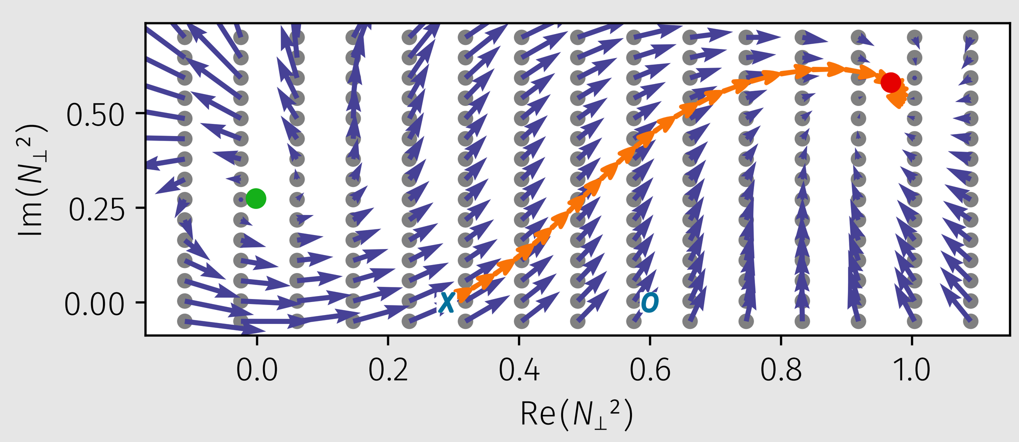
How to solve the dispersion relation for **relativistic** hot plasma?

$$\Lambda(\mathbf{x},\mathbf{N}) = \det\left[\mathbf{N}\otimes\mathbf{N} - N^2 I + \tilde{\varepsilon}(\mathbf{x},\mathbf{N})\right] = 0, \qquad (1)$$

where I identity, $\mathbf{x}$ position, $\mathbf{N}$ refractive index and $\tilde{\varepsilon}$ AC dielectric tensor.

Standard method: rewrite eq. 1 *formally* as $\Lambda(N_\perp^2) = AN_\perp^4 + BN_\perp^2 + C = 0$, substitute $N^2_{\perp,\text{cold}}$ as a first guess, solve using quadratic formula, iterate until $N_\perp^2$ converges.

Issues: slow convergence or failure when multiple roots approach each other [2]; works only for X, O mode solutions.



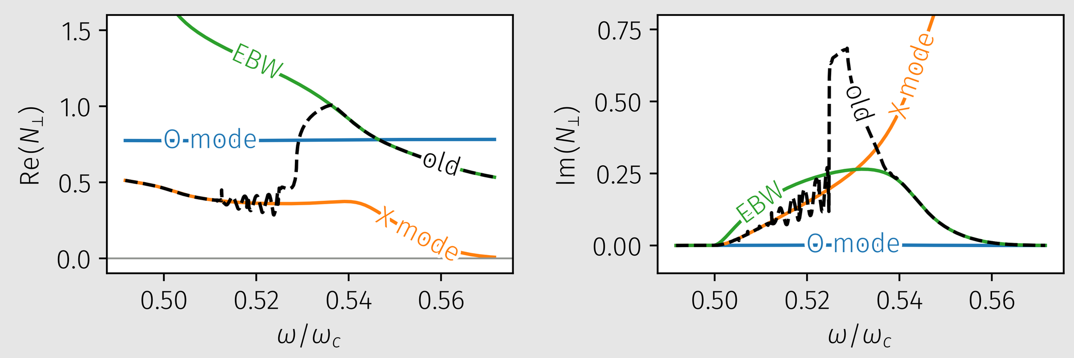
Iterative method **stepping** from the cold **X** approx., towards the **wrong**, instead of the **true** root. ($N_\parallel \approx 0$, $\omega_p^2/\omega^2 = 0.4$, 2nd harmonic)

New dispersion solver

Advantages: always **stable**, finds **all solutions**, including Electron Bernstein Waves (EBW).

How it works: exploit algebraic properties of dispersion relation

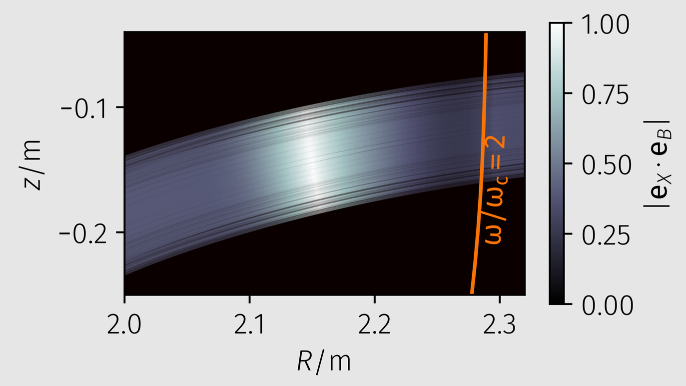
1. Farina formulation [3]: $\tilde{\varepsilon}(\mathbf{N})$ → single power series in $z = N_\perp^2$ → degree $l-1$ polynomials (l, highest harmonic)
2. eq. 1 → $\Lambda(z) = \sum_{k=1}^{3l-2} p_k z^k = 0$
3. find all roots simultaneously using Aberth method [4]



Old (standard) vs new solver near a branch point. The old solver is destabilised and jumps branche, the new finds all branches correctly.

Mode conversion?

The new solver allows detecting mode conversions events:



X-B coupling strength at 2nd harmonic estimated via the polarisation vectors $\mathbf{e}_X \cdot \mathbf{e}_B$

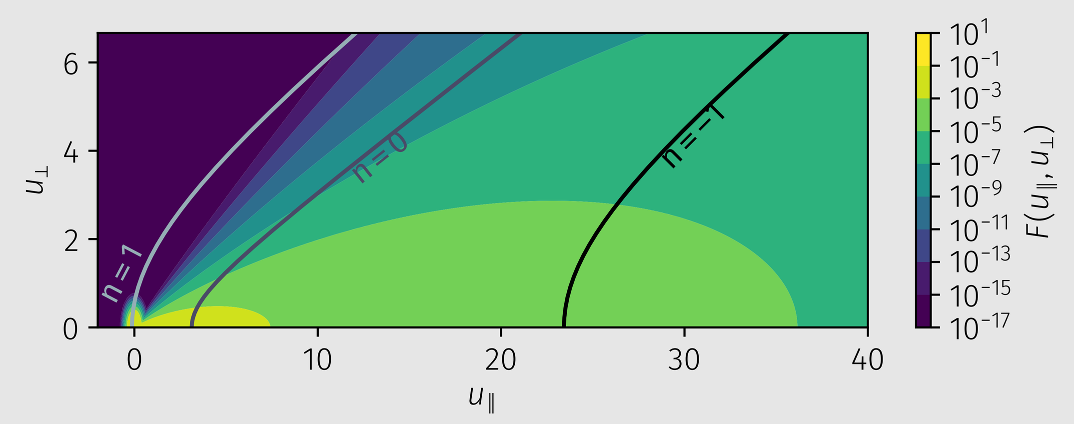
In combination with ray-splitting [5], could allow studying O-X-B schemes [6] or X-B conversion during the plasma start-up.

Under investigation!

Modelling runaway-driven instabilities

Runaways Electrons (RE) can drive kinetic instabilities: both Čerenkov and anomalous Doppler (AD) resonances contribute to pitch-angle scattering and lowering the energy of the RE [7].

$$\omega\gamma(1 - N_\parallel\beta_\parallel) = n\omega_c \qquad \text{with } n \leq 0, |N_\parallel| > 1$$



Resonance curves for $N_\parallel > 1$ overlaid the electrons distribution function

This mechanism can give information on the RE distribution: RE → plasma waves (instability) → EM waves → antenna [8].

Three effects to consider:

1. damping by **EC resonance** with bulk electrons
2. damping by electron-ion **collisions**
3. amplification by **AD resonance** with RE beam.

1. EC damping

As for ECRH: compute the dielectric tensor $\tilde{\varepsilon}$ → solve the dispersion relation → $\mathrm{Im}(N_\perp)$ → absorption coeff. α.

$\tilde{\varepsilon}$ involves Landau integrals of the form [3]:

$$Q_{hm}(n) \propto \int_{-\infty}^{+\infty} du_\parallel \int_{\gamma_\parallel}^{+\infty} d\gamma \frac{u_\parallel^h(\gamma^2-\gamma_\parallel^2)^m}{\gamma-\gamma_n} e^{-\mu\gamma}$$

where $u_\parallel$, $\gamma_\parallel$ momentum and Lorentz factor, $\gamma_n = N_\parallel u_\parallel + n(\omega_c/\omega)$, m order of FLR expansion, $h = 0, 1, 2$ and $n = 0, \pm 1, \cdots \pm m$ harmonics.

The case $|N_\parallel| > 1$ (required for instability!) **is tricky**:

- For $\mathrm{Im}\, Q_{hm}(n)$ → evaluate modified spherical Bessel i_m. New routine combining recurrence, Laurent and asymptotic series – depending on $N_\parallel$ – to avoid **numerical instabilities**.
- For $\mathrm{Re}\, Q_{hm}(n)$ → **careful** numerical integration: Q_{h0} has logarithmic **singularities** and Q_{hm} with $m > 0$ is *almost* discontinuous at resonances $\gamma = \gamma_n$.

2. Collisional damping

Modelled by adding a frictional force to the electron momentum equation:

$$\frac{d\mathbf{u}_e}{dt} = -\mathbf{u}_e\nu, \qquad \text{with } \nu \text{ the e-i collisional frequency.}$$

ν/ω is small ($\approx 10^{-8}$) → small anti-hermitian $\tilde{\varepsilon}_A$ → $\mathrm{Im}(N_\perp)$ and absorption coeff.

Simple expression obtained via **perturbation theory**.

3. Amplification by RE

Which runaway distribution?

1. ~~Lorentz-boosted Maxwellian~~ [9] → less absorption but **no amplification** found
2. Exponential with pitch-angle spread [10]

$$F(u,\theta) \propto u^{-2}\exp\left(-\frac{u}{u_0} - \frac{\theta^2}{\theta_0^2}\right)$$

Small RE fraction n_b/n_e → small $\tilde{\varepsilon}_A$ → α via perturbation theory:

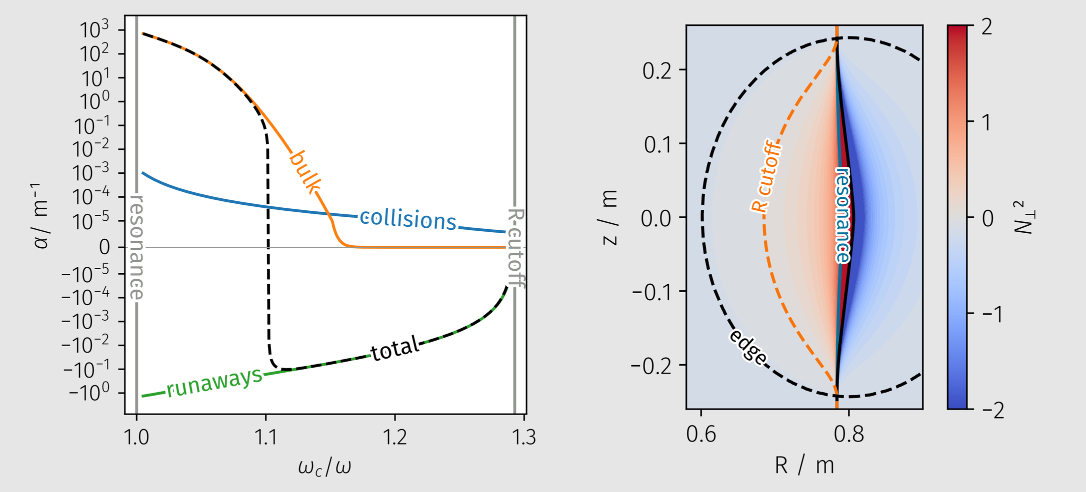
$$\alpha = -k_0\left(\frac{|\tilde{e}|^2}{Z_0|\tilde{S}|}\right)\frac{\pi}{|N_\parallel|}\frac{\omega_p^2}{\omega^2}\frac{n_b}{n_e}\sum_n\int u[Q_nR_n]_{\theta=\theta^*(u)}du$$

where Q_n depends on polarisation, R_n on RE distribution, θ^* is the resonant pitch-angle.

$\rho_L \approx 0$ approx is not good → full **numerical integration** and sum of $n = 0, \pm 1, \pm 2, \ldots$ harmonics ($n = 0, -1$ dominate, as expected!)

Stability analysis

- Slow X wave: f = 40 GHz, $N_\parallel$ = 1.05
- TCV-like plasma: $n_e \approx 1\times 10^{18}$ m^{-3}, $T_e \approx 0.25$ keV
- Runaways: E_b = 10 MeV, $n_b/n_e \approx 0.05$.



(a) absorption v amplification (b) region of wave propagation

⟶ **amplification** is possible!

Next steps: perform raytracing, study the spectrum

Summary

We have presented the ongoing efforts to extend GRAY to new applications:

- the new solver for relativistic dispersion relation solves stability issues of and unlocks new possibilities for describing X-B conversion
- the early results in modelling the interaction between RE and EC waves are promising and show that RE-driven instabilities can be successfully simulated and reproduced under realistic plasma conditions.

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