Extending the GRAY beam-tracer beyond the standard electron cyclotron heating modelling

Michele Guerini Rocco[a,*], Lorenzo Figini[b], William Bin[b], Carmine Castaldo[c], Paolo Buratti[d,f], Alessandro Cardinali[e], Francesco Napoli[f]

[a] *Department of Physics, University of Milano-Bicocca, Milan, 20126, Italy*
[b] *Institute for Plasma Science and Technology, National Research Council, Milan, 20125, Italy*
[c] *Consorzio RFX, Padua, Italy*
[d] *IAPS - INAF Institute for Space Astrophysics and Planetology, Rome, Italy*
[e] *CNR, Institute for Complex Systems, Polytechnic University of Turin, Turin, Italy*
[f] *ENEA, Fusion and Nuclear Safety Department, Frascati, Rome, Italy*

1. Introduction

Electron Cyclotron (EC) waves are playing an increasingly important role in the operation of next-generation fusion devices: Electron Cyclotron Resonant Heating (ECRH), will be the main plasma heating mechanism, and Electron Cyclotron Current Drive (ECCD) will be fundamental in controlling instabilities like sawteeth and Neoclassical Tearing Modes (NTM) [1]; in superconducting devices, EC waves are also becoming an essential tool to assist the ohmic breakdown in the start-up phase [2]; finally, the interaction between EC waves and runaway electrons is being investigated as a new promising channel for exploring runaway dynamics [3]. GRAY [4] is a well-established beam-tracing code and a valuable tool for modeling EC waves, whose use has so far been mostly limited to predictive modeling of standard ECRH scenarios. More recently, efforts have been made to extend the applicability of GRAY across several fronts.

2. New solver for the relativistic dispersion relation

The beam power absorption in GRAY is computed by solving – at each step of the raytracing – the dispersion relation for EC waves in a relativistic Maxwellian plasma:

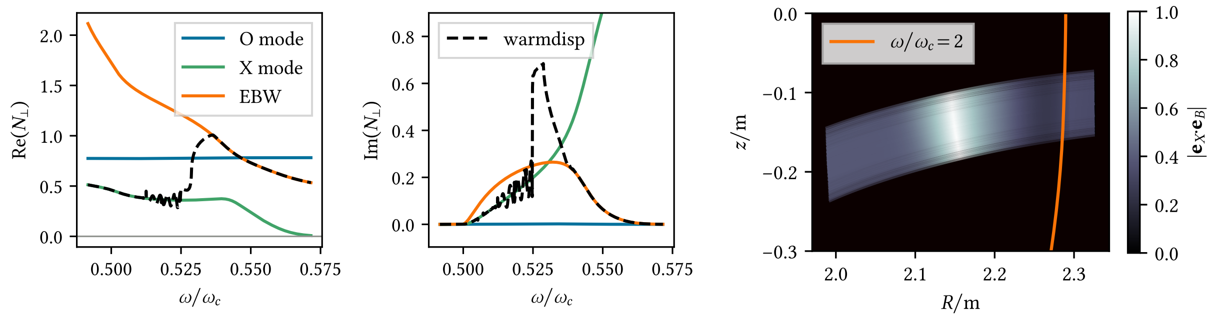
$$\Lambda(\mathbf{x}, \mathbf{N}) = \det\left[\mathbf{N} \otimes \mathbf{N} - N^2 I + \tilde{\varepsilon}(\mathbf{x}, \mathbf{N})\right] = 0, \tag{1}$$

where I is the identity, $\mathbf{x}$ the position, $\mathbf{N} = c\mathbf{k}/\omega$ the refractive index and $\tilde{\varepsilon}(\mathbf{x}, \mathbf{N})$ the AC dielectric tensor. The standard iterative technique of solving eq. 1 has stability issues: its convergence can slow down and even diverge away when multiple roots approach each other [5]. For example, near the upper-hybrid or at the 2nd harmonic, where mode conversion between

*Corresponding author.
Email address: `michele.guerinirocco@unimib.it` (Michele Guerini Rocco)

X branch and electromagnetic Electron Bernstein Wave (EBW) is possible. The method is also limited to solving for O or X branches, one at a time. To addresses these issues, we developed a new method that exploits the algebraic structure of eq. 1. In fact, using the Farina formulation [6], $\tilde{\varepsilon}(\mathbf{x}, \mathbf{N})$ is expressed as a single power series in $z = N_\perp^2$ and reduces to a polynomial of degree $l-1$, with l the highest possible harmonic. So, solving eq. 1 reduces to finding roots of the polynomial $\Lambda(z) = \Sigma_{k=1}^{3l-2} p_k z^k$, for which a great deal of algebraic methods are available. The newly written `hotdisp` subroutine thus computes the expansion up to l, performs polynomial operations to obtain the p_k and using Aberth method [7] estimates all complex roots of $\Lambda(z)$.

In fig. 1, results obtained with `hotdisp` relative to a DTT X2 heating scenario which happens to be close the X-B branch point ($T_e \approx 10\,\text{keV}$, $n_e = 1.5 \times 10^{19}\,\text{m}^{-3}$, $f = 170\,\text{GHz}$) are shown.



(a) (b)

Figure 1: fig. 1a, dispersion curves with the standard `warmdisp` solver (dashed) exhibiting unstable behaviour and jumping to the wrong branch; the solid lines are obtained by `hotdisp`. fig. 1b X-B conversion region visualised by a beam and the coupling strength between the mode polarisations vectors $\mathbf{e}_X \cdot \mathbf{e}_B$, obtained with `hotdisp`. The latter allows detecting mode conversion events and, if combined with a ray-splitting method [8], could be used to study O-X-B heating schemes [9] or X-B conversion during the plasma start-up. This possibility is currently being explored.

3. Modelling runaway-driven instabilities

Runaway electrons (RE) in tokamaks can drive kinetic instabilities. Both the Čerenkov and the anomalous Doppler (AD) resonances contribute to pitch-angle scattering and lowering the energy of RE beams [10]. This mechanism provide insights into the RE distribution: plasma instabilities can couple to electromagnetic waves that have been detected experimentally [3]. While, so far, modelling has mostly concerned with waves in the sub-GHz spectrum (whistler, lower-hybrid, Ion Bernstein Waves etc.) [11,12], here we explore the electron cyclotron (EC) range. The stability analysis of EC waves requires considering three effects: damping by EC resonance with bulk electrons, damping by electron-ion collisions and, most importantly, amplification by AD resonance with the RE beam.

The first is already considered in GRAY, but the capability of simulating propagation with $|N_\parallel| > 1$ has been newly implemented for this work. This condition is, in fact, necessary for the AD resonance: $\omega\gamma(1 - N_\parallel\beta_\parallel) = n\omega_c$ where $n < 0$. The computation of the relativistic plasma dielectric tensor has now been extended also to cover this case. This computation involves complex Landau integrals of the form:

$$Q_{hm}(n) \propto \int_{-\infty}^{+\infty} du_\parallel \int_{\gamma_\parallel}^{+\infty} d\gamma \, \frac{u_\parallel^h(\gamma^2 - \gamma_\parallel^2)^m}{\gamma - \gamma_n} e^{-\mu\gamma} \tag{2}$$

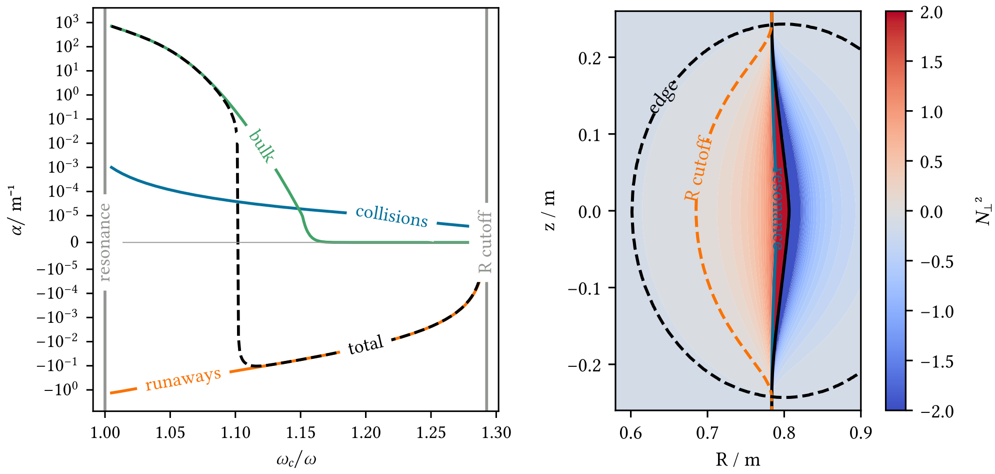
where $\gamma_\parallel = \sqrt{1 + u_\parallel^2}$, $\gamma_n = N_\parallel u_\parallel + n(\omega_c/\omega)$, m is the order of the Larmor radius expansion, $h = 0, 1, 2$ and $n = 0, \pm 1, \cdots \pm m$ are the harmonics. The imaginary part of $Q_{hm}(n)$ can be expressed as a combination of the modified spherical Bessel functions $i_m(x/2)$[1]. As mentioned in [6], these can be evaluated using a recurrence relation[2], however becoming numerically unstable for small $N_\parallel$ and overflowing when $|N_\parallel| \to 1$, which is our case. The subroutine has thus been rewritten to automatically switch between recurrence, Laurent and asymptotic series depending on the value of $N_\parallel$, keeping the absolute error below 10^{-10} for the first 20 harmonics. For the real part, a numerical integration of eq. 2 is necessary. The case $|N_\parallel| \geq 1$ is again non-trivial: Q_{h0} has logarithmic singularities and for $m > 0$ the integrand, though continuous, changes very rapidly, so the integration domain has to be split accordingly.

The second effect, instead, is easily modelled by adding a frictional force to the electron momentum equation: $d\mathbf{u}_e/dt = -\mathbf{u}_e\nu$, where ν is the electron-ion collisional frequency. Since ν/ω is small (10^{-8} typically), the anti-hermitian contribution to the dielectric tensor is small and the resulting $\mathrm{Im}(N_\perp)$, hence the absorption coeff., can be obtained via perturbation theory.

For the third and most important effect, we have investigated describing the RE as a relativistic streaming Maxwellian and obtaining the combined dielectric tensor by a Lorentz transformation [13]. However, the instability drive never overcomes the EC absorption using this distribution. Nevertheless, promising results have been obtained with an exponential $F(u) \propto u^{-2}\exp(-u/u_0)$ and a small spread in pitch-angle, as proposed in [11]. Since the RE density is small, we again employ perturbation theory to obtain the absorption coeff. In fig. 2, the preliminary results for the stability analysis in a TCV-like plasma scenario are reported.

[1] The argument of the functions is $x = \mu N_\parallel \sqrt{(n\omega_c/\omega)^2 - (1 - N_\parallel^2)}/(1 - N_\parallel^2)$.

[2] This is the analogous of Miller's recurrence algorithm in the spherical case.



(a) (b)

Figure 2: Preliminary results for the stability analysis of slow X waves with $f = 40$ GHz, $N_\parallel = 1.05$ in a TCV-like plasma scenario ($\omega_p^2/\omega^2 \approx 0.03$, $T_e \approx 0.25$ keV, 10 MeV RE beam with density $n_b/n_e \approx 0.05$). In fig. 2a the wave absorption coeff. is shown, including the separate contributions from collisions, bulk and RE. The region with $\alpha < 0$ is where AD dominates and the kinetic instability takes place. In fig. 2b, a contour plot of $N_\perp^2$ (cold plasma) showing the region of wave propagation, which is between the $n = 1$ resonance and the R cutoff (here on the h.f.s. because $N_\parallel > 1$), is shown.

4. Summary

We have presented the ongoing efforts to extend GRAY to new applications: the new solver for the relativistic dispersion relation addresses the issues of the standard method and unlocks new possibilities for describing X-B mode conversion; the early results in modelling the interaction between RE and EC waves are promising and show that RE-driven instabilities can be successfully simulated and reproduced under realistic plasma conditions.

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