Confinement Scaling with Machine Size in the Updated ITPA Global H-Mode Energy Confinement Database

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** See the authorlist of C. Maggi et al., Nucl. Fusion 64, 112012 (2024)*
*** See the authorlist of E. Joffrin et al., Nuclear Fusion 64, 112019 (2024)*

1 Background & motivation

The IPB98(y, 2) scaling law was derived in 1999 [1].

$$\tau_E^{\mathrm{IPB98(y,2)}} = 0.056 I_p^{0.93} B_t^{0.15} \overline{n}_e^{0.41} P_{\mathrm{l,th}}^{-0.69} R_{\mathrm{geo}}^{1.97} \kappa^{0.78} \varepsilon^{0.58} M_{\mathrm{eff}}^{0.19}$$

Some 23 years later, an updated regression on the expanded version of the same database showed [2]:

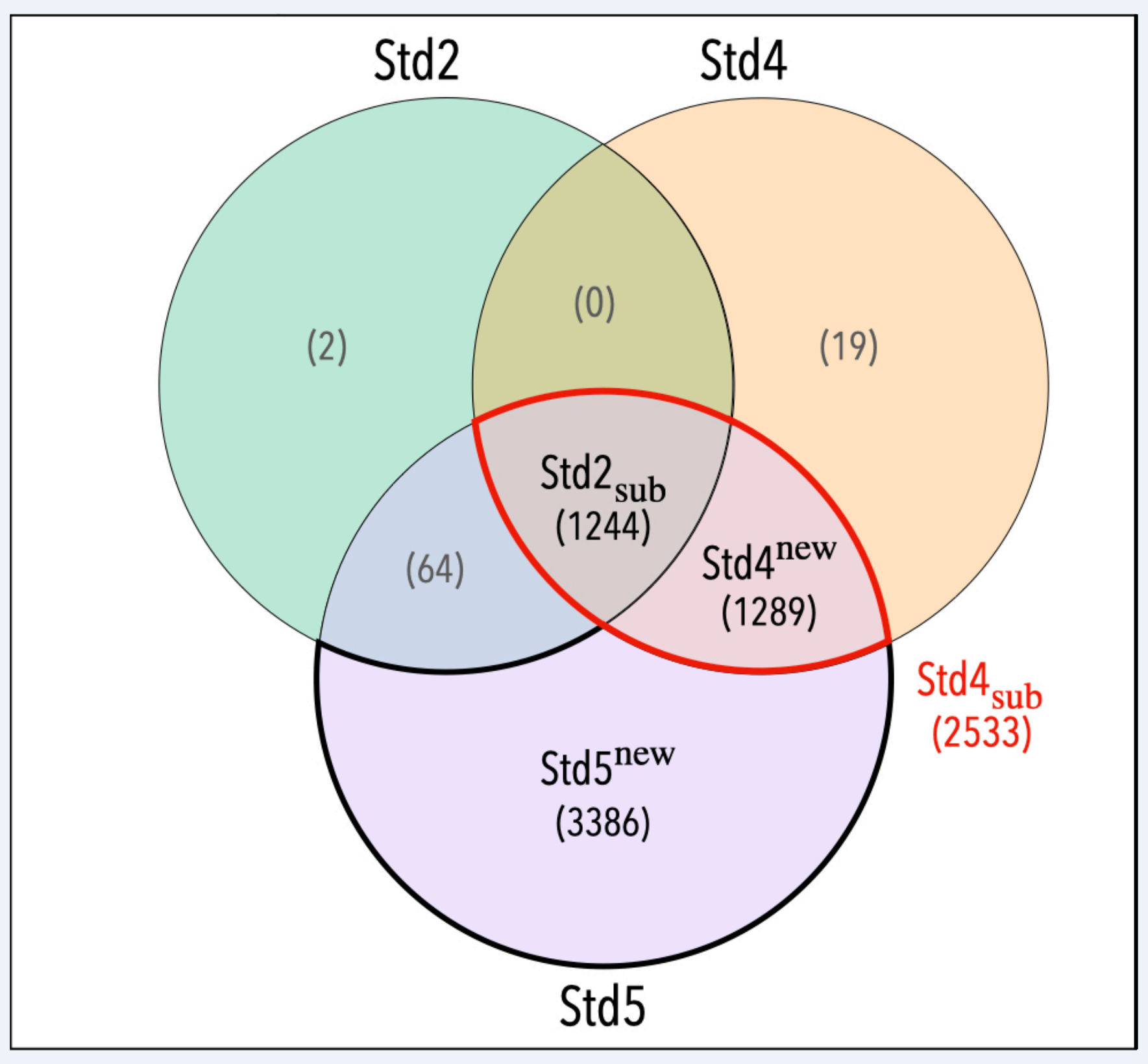
$$\tau_E^{\mathrm{ITPA20}} \propto R_{\mathrm{geo}}^{1.71\pm0.32} \qquad \tau_E^{\mathrm{ITPA20-IL}} \propto R_{\mathrm{geo}}^{1.19\pm0.27}$$

The size exponent α_R ***weakened***.
Two candidate explanations were investigated:
(i) Statistical artefacts, in particular multicollinearity
(ii) Physical properties of the additional discharges

2 ITPA H-mode database

Three database iterations are used:

$\mathrm{Std2_{sub}}$: 1244 pts (8 machines)	$\alpha_R = 1.7$	(2.1)	[1]
$\mathrm{Std4_{sub}}$: 2533 pts (13 machines)	$\alpha_R = 1.4$	(2.0)	[3]
Std5: 5919 pts (14 machines)	$\alpha_R = 1.3$	(1.6)	[2]



Spherical devices excluded (distinct transport physics).

Leading to a reduced range in ε

Hence, ε was removed from the regression model.

3 Multicollinearity diagnostics

Each regression coefficient has an associated standard deviation which is governed by:

- The model
- Predictors range
- Multicollinearity
- Measurement uncertainty

$$\sigma_i \equiv \sqrt{\mathrm{Var}\left(\hat{\beta}_i\right)} = \sqrt{\frac{\sigma_\epsilon^2}{\mathrm{SS}_i} \cdot \mathrm{VIF}_i}, i = 1, \ldots, p$$

Variation inflation factors (VIF)

Variable	$\mathrm{Std2_{sub}}$	$\mathrm{Std4_{sub}}$	Std5
I_p	7.0	6.9	7.8
R_{geo}	5.1	5.0	6.6

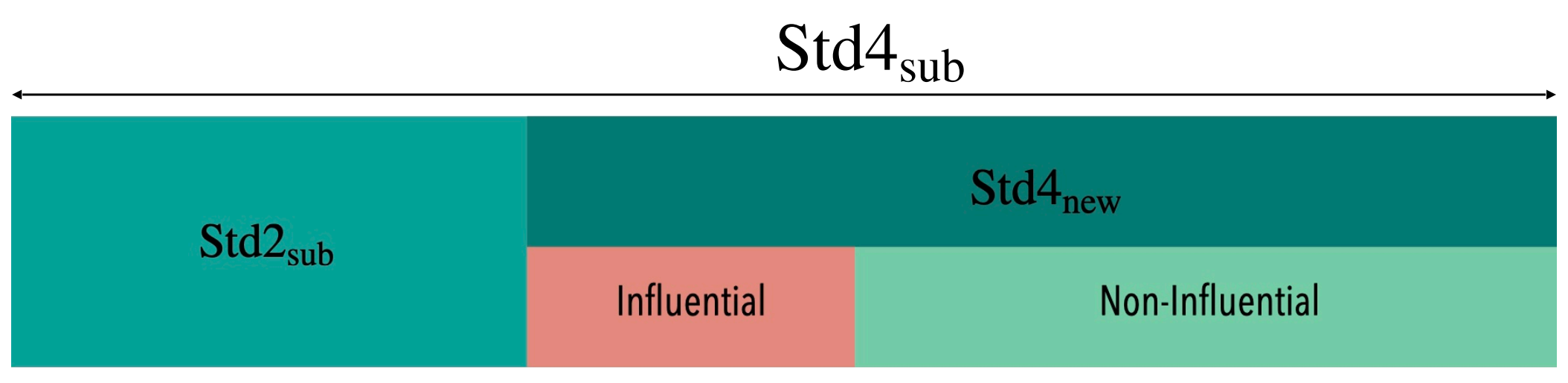
VIF > 5 are indicative of strong multicollinearity

- While multicollinearity is pronounced, the effect on coefficient variance remains limited
- ***We deduce that multicollinearity is not the sole cause of reduced size scaling.***

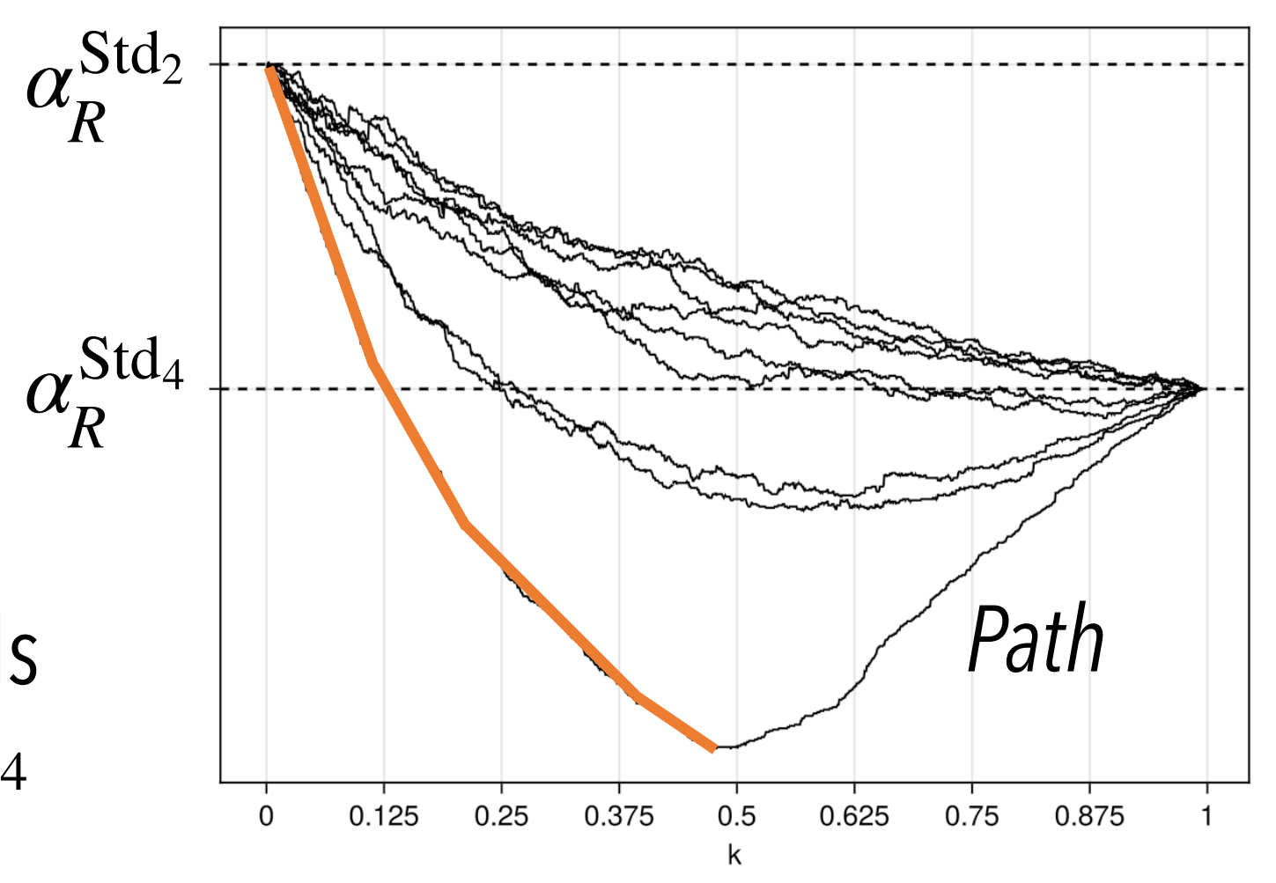
4 Identification of the Influential Set

To avoid confusion of abstraction: *we explain the optimisation procedure by focusing on the example of the transition from $\mathrm{Std2_{sub}}$ to $\mathrm{Std4_{sub}}$. The transition of $\mathrm{Std4_{sub}}$ to Std5 follows analogously.*

1 ORDER

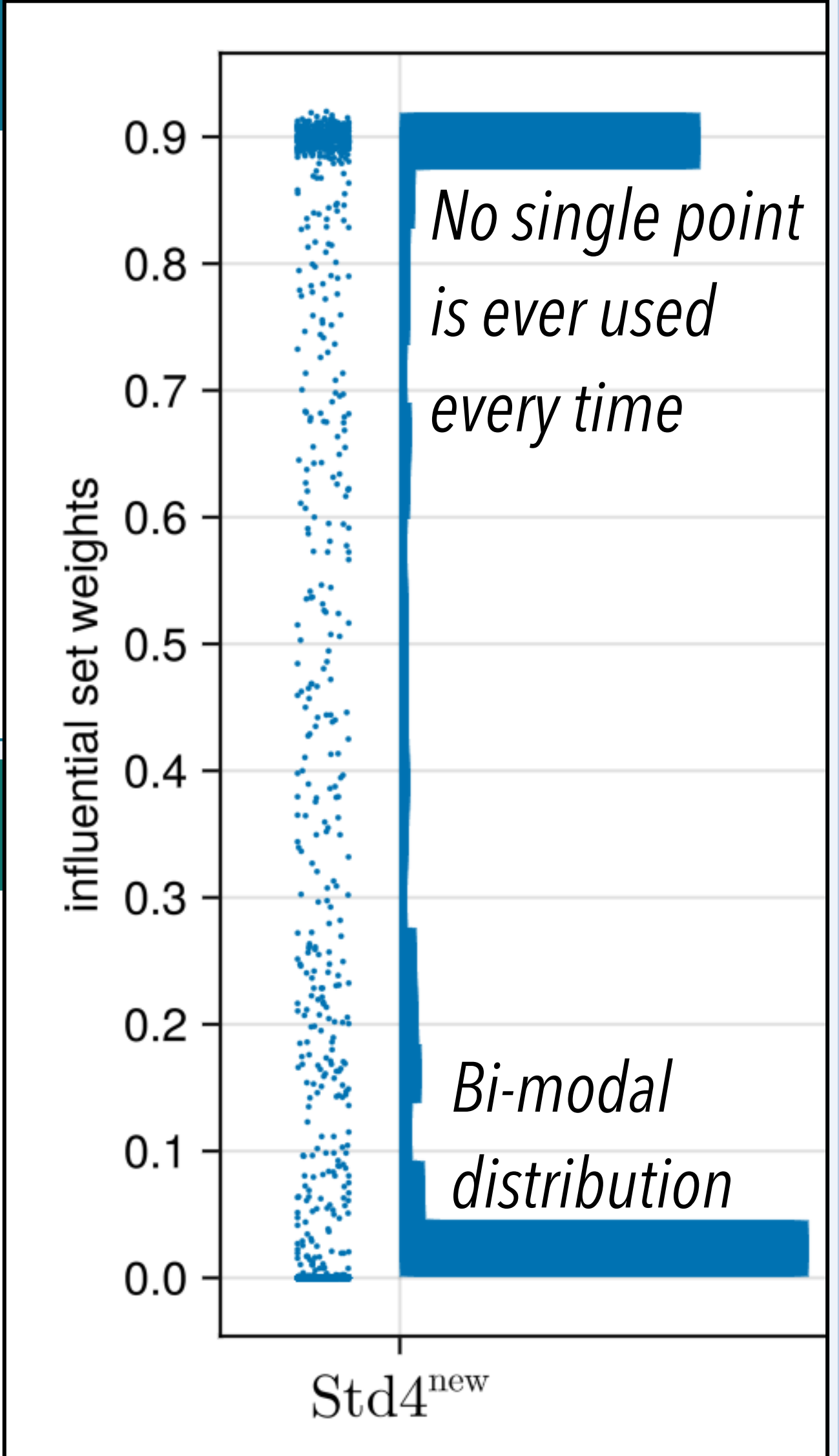


- Aim: determine the smallest subset of $\mathrm{Std4_{new}}$ that results in the largest reduction in α_R (influential set)
- Iteratively add points from $\mathrm{Std4_{new}}$ into $\mathrm{Std2_{sub}}$
- Perform a regression in each instance, recording α_R
- Optimal ordering of points leads to an intermediate $\alpha_R < \alpha_R^{\mathrm{Std_4}}$

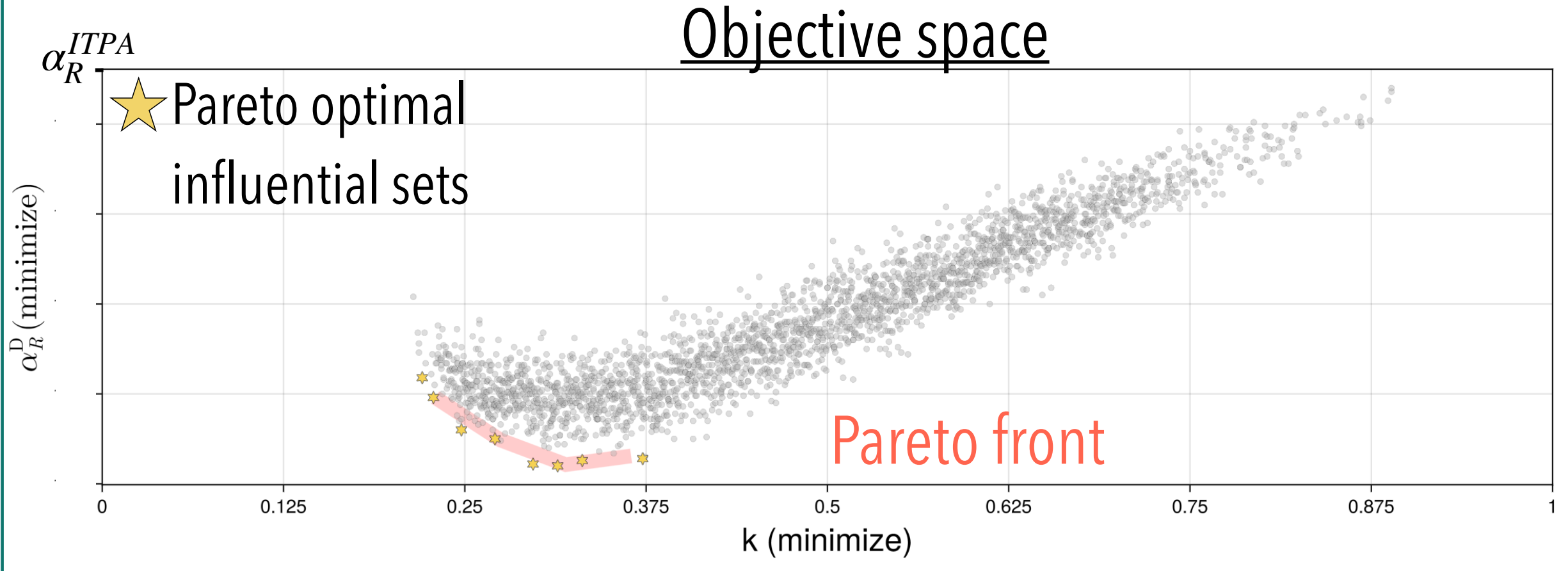


2 Quantify

- For each path generated
  - Assign 1 to each point leading to the minimum of the path (highlighted in orange)
  - Assign 0 otherwise
- Average over the number of paths generated
- Consider the mean value of each data-point a probability of inclusion in the influential set

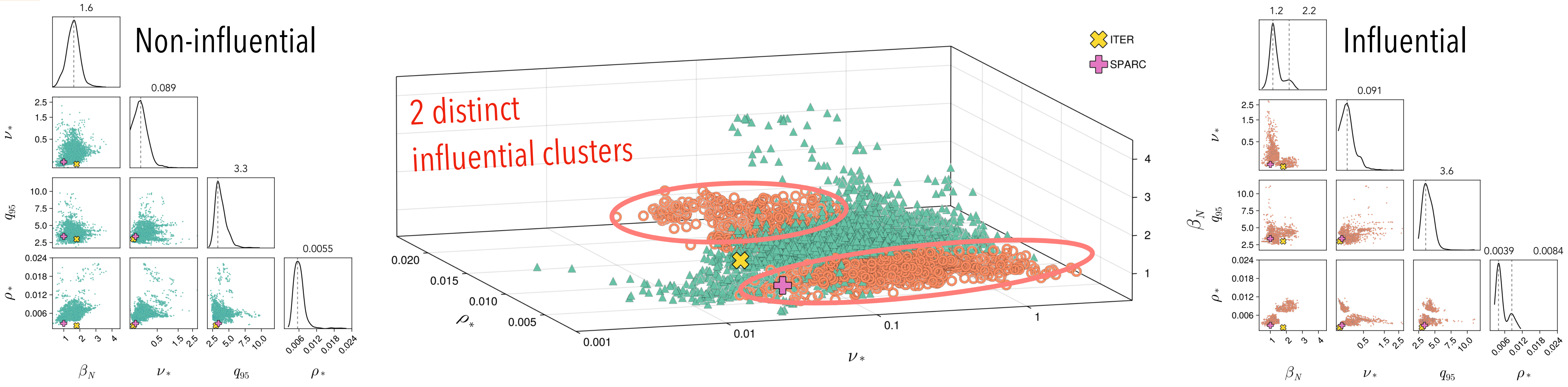


3 Optimise



- Extract subsets of size k through weighted sampling.
- Evaluate based on α_R^D and subset size, k.

4 Classify



5 Two distinct scaling laws

Non-Influential Scaling (α_R = 1.7)

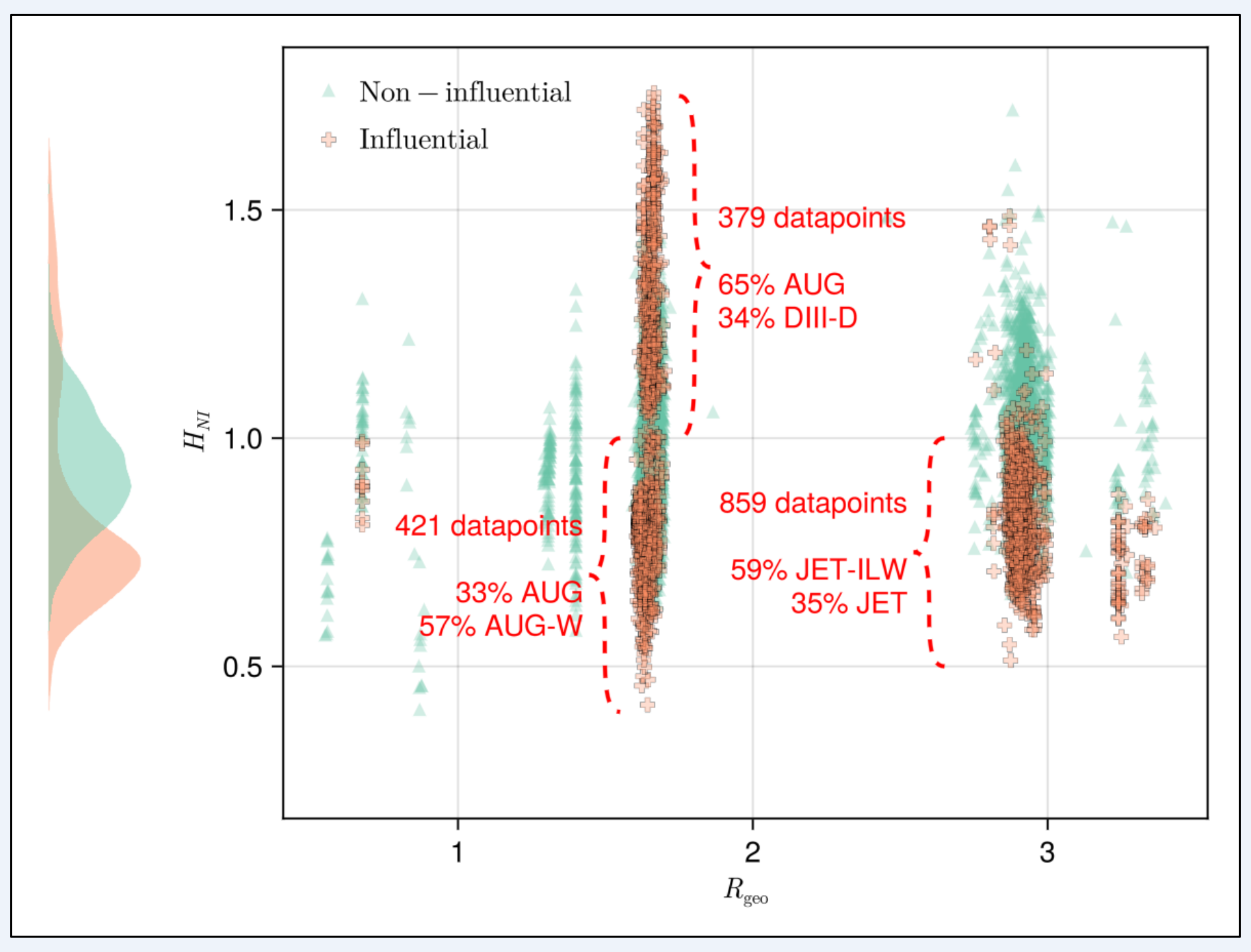
$$\tau_{\mathrm{E,th}}^{\mathrm{NI}} = 0.04 I_{\mathrm{p}}^{1.1} B_{\mathrm{t}}^{0.065} \overline{n}_{\mathrm{e}}^{0.35} P_{l,th}^{-0.71} R_{\mathrm{geo}}^{1.7} \kappa^{0.77} M_{\mathrm{eff}}^{0.14}$$

RMSE = 0.14 · Regained size scaling · Negligible field scaling (α_B = 0.065) · Strong power degradation $\alpha_P = -0.71$

Influential Scaling (α_R= 0.56)

$$\tau_{\mathrm{E,th}}^{\mathrm{I}} = 0.177 I_{\mathrm{p}}^{1.5} B_{\mathrm{t}}^{-0.25} \overline{n}_{\mathrm{e}}^{-0.19} P_{l,th}^{-0.62} R_{\mathrm{geo}}^{0.56} \kappa^{0.73} M_{\mathrm{eff}}^{0.54}$$

RMSE = 0.15 · Weak size scaling · Negative density exponent ($\alpha_n = -0.19$) · **Descriptive role only**



6 Conclusions

1. Multicollinearity is present but cannot solely account for α_R reduction
2. Maximally influential set has been determined with an optimisation procedure
3. Influential set well localised in dimensionless ($\rho^* \cdot \nu^* \cdot \beta_N \cdot q_{95}$) space, forming two clusters
4. Virtually all observations excluded from the database to form the ITER-like subset used for deriving ITPA20-IL scaling are assigned to the non-influential subset
5. Non-influential scaling recovers IPB98-like size dependence α_R = 1.7.

ITER & SPARC: 84% non-influential probability
→ NI scaling is the relevant prediction

Tokamak	$\tau_{\mathrm{E,th}}^{\mathrm{I}}(s)$	$\tau_{\mathrm{E,th}}^{\mathrm{NI}}(s)$	$\tau_{\mathrm{E,th}}^{\mathrm{reference}}(s)$
ITER	1.9	3.8	3.6
SPARC	0.46	0.88	0.77

References
[1] ITER Physics Basis Editors, Nucl. Fusion 39, 2175 (1999)
[2] G. Verdoolaege et al., Nuclear Fusion 61, 076006 (2021)
[3] D.C McDonald et al., Nuclear Fusion, 47, 147 (2007)
[4] J. Hall et al., Nuclear Fusion (2026) (under review)
[5] C. Angioni et al., Nuclear Fusion 66 026006 (2026)

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