A coherent-mode-based criterion for diffusion-dominated transport of runaway electrons during disruptions

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Abstract

Runaway electron (RE) transport during tokamak disruptions is critically influenced by magnetic perturbations, yet the dominant transport mode—diffusive or convective—remains insufficiently quantified. We propose a new coherent-mode-based indicator, the coherent mode-dominance factor Θ, to determine the transport regime directly from MHD magnetic field spectra. This factor combines the amplitude dominance and phase coherence of the magnetic perturbations. Its predictive power is validated against the Chirikov overlap parameter (for magnetic stochasticity) and the Péclet number (from test-particle simulations) using three representative JOREK disruption scenarios. A clear mapping is established: $\Theta < 0.5$ corresponds to diffusion-dominated transport (low Pe), while $\Theta > 0.5$ indicates convection-dominated transport (high Pe). The proposed framework provides a fast, simulation-free tool to assess the effectiveness of magnetic perturbations in exhausting seed REs, with direct implications for disruption mitigation.

1 Introduction

Tokamak disruptions can generate multi-MeV runaway electrons (REs) that carry a substantial fraction of the plasma current, posing a serious threat to plasma-facing components [1]. Enhancing radial transport of REs via magnetic perturbations—either externally applied or arising from MHD instabilities during massive material injection—is a promising mitigation strategy [2, 3]. In sufficiently stochastic magnetic fields, RE transport becomes diffusive, and the seed population can be exhausted before avalanching into a dangerous beam [4]. However, when the perturbation spectrum is dominated by a few coherent modes, ordered magnetic structures can guide particles along field lines, leading to convection-dominated transport. Quantifying the competition between these two regimes is essential for designing reliable disruption mitigation schemes.

The Chirikov overlap parameter S_{Chir} is commonly used to assess magnetic stochasticity [5]: $S_{\mathrm{Chir}} > 1$ indicates island overlap and global stochasticity, favoring diffusion. Yet S_{Chir} provides no information about convective transport driven by coherent modes. The Péclet number $Pe = L|V|/D$, extracted from test-particle simulations [3], directly compares convection to diffusion, but is computationally expensive and does not reveal the underlying magnetic structure responsible for the observed transport.

In this work, we introduce a new dimensionless parameter, the coherent mode-dominance factor Θ, which quantifies the degree to which a few coherent modes dominate the magnetic perturbation spectrum. Θ can be computed solely from the MHD magnetic field, without the need for particle simulations. We apply Θ to three characteristic disruption scenarios simulated by JOREK [6] and validate its predictions against both S_{Chir} and Pe. A clear correlation between Θ and the transport regime is demonstrated.

2 Numerical setup and analysis tools

2.1 Disruption scenarios and particle simulations

Three time slices from JOREK nonlinear MHD simulations of an ITER H-mode thermal quench mitigated by neon-doped shattered pellet injection (SPI) are analyzed [6]. They represent different degrees of magnetic stochasticity (see Table 1). For each case, the PTC guiding-center code [4,7] is used to track 5 MeV seed REs with pitch $p_{\|}/p = 0.9$. The simulation time (3×10^7 steps, $\Delta t = 5.7 \times 10^{-10}$ s) is sufficient to reach a steady-state transport regime.

Table 1: Selected time slices from JOREK fluid simulation.

Fluid Case	**Notation**	**Characteristic**	**Time (ms)**
DH-QP-dt1	A	Strong stochastic magnetic field	1.839
DH-FP-dt0	B	Medium stochastic magnetic field	2.069
DH-QP-dt0	C	Inner flux surfaces closed	6.116

2.2 Chirikov overlap parameter and Péclet number

The Chirikov overlap parameter for neighboring poloidal modes m and $m+1$ is defined as $S_{\text{Chir}} = \frac{\Delta\psi_{m,n}+\Delta\psi_{m+1,n}}{2|\psi_{m+1,n}-\psi_{m,n}|}$, where $\Delta\psi_{m,n}$ is the island width and $\psi_{m,n}$ the resonant surface. $S_{\text{Chir}} > 1$ means overlapping occurs between islands, leading to diffusive transport.

The Péclet number is given by $Pe = (r_a - r)\,|V(r)|/D(r)$, where r_a is the minor radius. $Pe < 1$ corresponds to diffusion dominance, $Pe > 1$ to convection dominance.

2.3 Coherent mode-dominance factor

The perturbed poloidal flux is Fourier decomposed as $\tilde{\Psi}(r,\theta,\phi) = \sum_{m,n} \Psi_{m,n}(r)e^{i(m\theta-n\phi)}$. The normalized amplitude of mode (m,n) at a given radial position is defined as

$$Theta(\psi) = \Xi(\psi)\cdot C_{\text{top}}(\psi) = \sum_{(m,n)\in\mathcal{M}_{\text{top}}} \Phi_{m,n}(\psi)\cdot \frac{\left|\sum_{\mathcal{M}_{\text{top}}} \Phi_{m,n}e^{i\theta_{m,n}}\right|}{\sum_{\mathcal{M}_{\text{top}}} \Phi_{m,n}}, \quad (1)$$

where the $\Phi_{m,n}(\psi) = \frac{m^2\Psi^2_{m,n}}{\sum_{m,n} m^2\Psi^2_{m,n}}$is mode perturbation contribution factor of mode (m,n) at flux ψ. We then identify the set $\mathcal{M}_{\text{top}}$ of the strongest modes such that the flux-surface-averaged cumulative amplitude satisfies $\langle\sum_{(m,n)\in\mathcal{M}_{\text{top}}} \Phi_{m,n}\rangle_\psi \geq 0.5$. The dominance factor is $\Xi(\psi)$), and the phase correlation factor is $C_{\text{top}}(\psi)$ The coherent mode-dominance factor is then defined as $\Theta(\psi) = \Xi(\psi)\cdot C_{\text{top}}(\psi)$. By construction, $\Theta \in [0,1]$. When $\Theta > 0.5$, a few modes with correlated phases dominate, fostering coherent magnetic structures that promote convective transport. When $\Theta < 0.5$, either the spectrum is broad (many modes contribute) or the dominant modes are phase-decorrelated, resulting in a stochastic, diffusion-dominated field.

3 Results and validation

3.1 Chirikov parameter and particle convergence

Figure 1(a) shows S_{Chir} and the particle convergence for Case A. $S_{\text{Chir}} > 1$ over most of the radius indicates strong stochasticity. The mean and variance of the OMP radius for particles started at different initial $\hat{\psi}$ converge within ~ 100 toroidal turns, confirming diffusive behavior. Case B shows similar but slower convergence, while Case C exhibits no convergence in the inner region ($\hat{\psi} < 0.5$), consistent with the low S_{Chir} there (not shown).

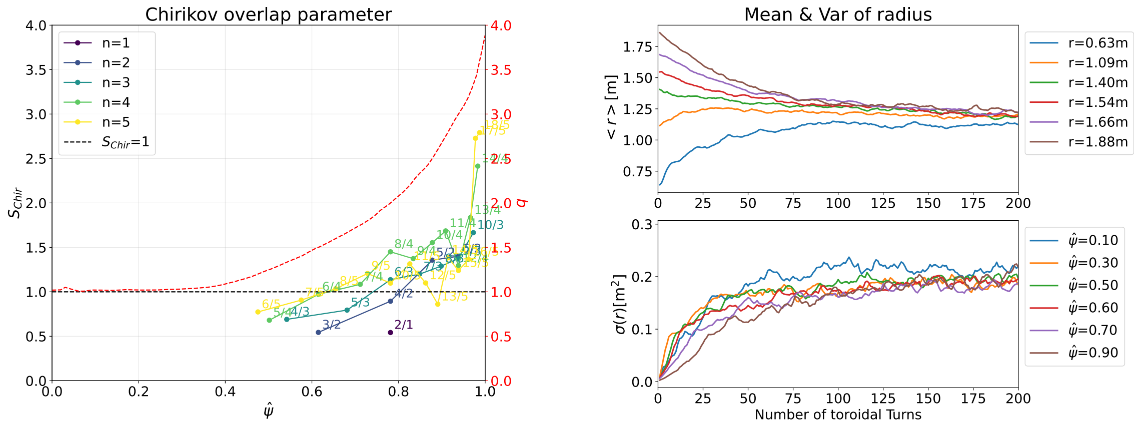

Figure 1: (a) Chirikov parameter profile for Case A. (b) Mean (top) and variance (bottom) of the OMP crossing radius vs. toroidal turns for particles started at different initial radii.

3.2 Coherent mode-dominance factor and Péclet number

Figure 2 shows $\Theta(\psi)$ and Pe for the three cases. Case C exhibits $\Theta > 0.5$ over a wide radial range ($0.2 < \hat{\psi} < 0.8$), predicting convection-dominated transport, which is corroborated by $Pe \sim 10$ in the inner region and $Pe > 1$ almost everywhere. Case A has $\Theta \ll 0.5$ and $Pe < 1$, indicating diffusion dominance. Case B is intermediate, with $\Theta < 0.5$ and $Pe \sim 1$, suggesting a competition between convection and diffusion. The characteristic loss time τ further supports these findings: $\tau_A = 66.78\,\mu\mathrm{s}$, $\tau_B = 86.04\,\mu\mathrm{s}$, and $\tau_C = 168.55\,\mu\mathrm{s}$. Also, the remaining particles number after simulation is much lower in case A. Thus, the diffusion-dominated cases exhaust seed REs faster, in line with the low Θ values.

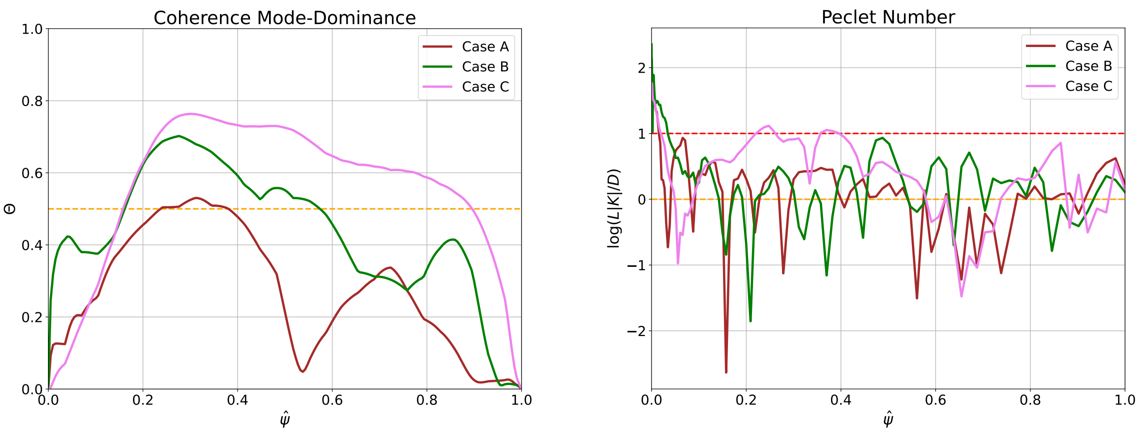

Figure 2: Radial profiles of (a) the coherent mode-dominance factor Θ and (b) the Péclet number Pe for the three cases. The dashed lines mark $\Theta = 0.5$ and $Pe = 1$.

3.3 Validation of the diffusion regime

For Case A ($\Theta < 0.5$), the particle density is expected to follow the eigenmode of the diffusion equation $r^{-1}\partial_r(rD\partial_r n) = \lambda n$ with $n(r_a) = 0$ and $\partial_r n(0) = 0$. Figure 3 (left) shows that the normalized density profiles at different times indeed consists with the 0-th order eigenmode (red dashed). Moreover, the real particle flux across flux surfaces agrees well with the diffusive flux $-D\nabla n$ (right). For the non-diffusive cases (B and C), neither the eigenmode nor the diffusive flux captures the particle behavior, confirming the transport regime predicted by Θ.

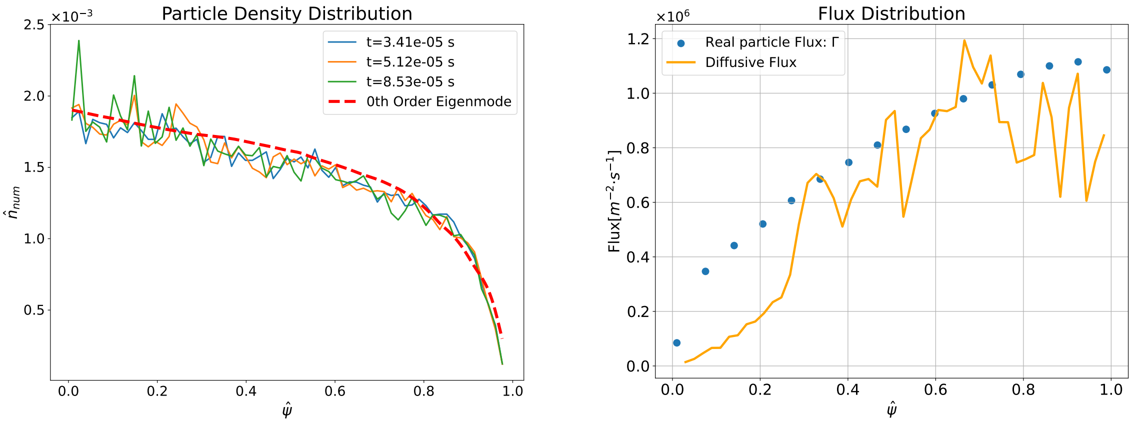

Figure 3: Validation for Case A. Left: normalized density profiles (blue, orange, green) converge to the eigenmode (red dashed). Right: real particle flux (blue dots) vs. diffusive flux (yellow).

4 Conclusion

We have introduced a novel coherent mode-dominance factor Θ that quantifies the competition between diffusion and convection in RE transport directly from the magnetic perturbation spectrum. Applied to three JOREK disruption scenarios, $\Theta > 0.5$ correctly identifies convection-dominated transport (high Pe, slow RE loss), while $\Theta < 0.5$ identifies diffusion-dominated transport (low Pe, fast RE loss). Compared to the Chirikov parameter, Θ additionally captures the influence of phase coherence, providing a more complete picture of the magnetic topology. Because Θ requires only field data, it offers a computationally efficient tool for evaluating magnetic perturbation strategies in disruption mitigation. Such a framework is essential for improving predictive models of runaway electron dynamics and for guiding the design of mitigation strategies that rely on controlled magnetic perturbations, which may pave the way toward benign termination.

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