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kobra: a new Vlasov code for sheath physics with AMR

Background

For the operation of tokamaks, it is fundamental to understand the interaction of the plasma with the reactor wall. In this region, i.e. the scrape off layer (SOL), particle distributions are often non-Maxwellian and particle density is sparse. Particle-in-cell codes offer a straight-forward formulation to addressing kinetic problems but in the SOL, this approach leads to statistical errors.

Our goal **is to develop a Vlasov code, *kobra*, devoid of these errors, applicable to plasma wall interactions equipped with AMR for computational speedup.**

Numerical Code

kobra is currently a second-order code which solves the Vlasov-Poisson system. We use an adaptive mesh routine for computational gain.

$$\frac{\partial f_s}{\partial t} + v\left(\frac{\partial f_s}{\partial x}\right) + \frac{q_s}{m_s}(v \times B_0 + E)\left(\frac{\partial f_s}{\partial v}\right) = 0$$

$$-\frac{\partial^2 \phi}{\partial x^2} = \sum_s n_s q_s \qquad E = -\nabla\phi$$

Vlasov Equation (hyperbolic solver):

- Runge-Kutta midpoint scheme for temporal discretization
- Fluxes are calculated with an upwind scheme with a van-Albada flux limiter [1]

Poisson Equation (elliptic solver):

- We generate a second *auxiliary grid* in the spatial coordinates to calculate the scalar potential
- We use a full approximation scheme (FAS) multigrid approach with a Gauss-Seidel smoother

Adaptive Mesh Refinement (AMR):

- We refine cells on a cell-by-cell basis
- We use the first two truncation error terms as our refinement criteria [2] following the architecture of [3]
- We employ defragmentation and load balancing routines to streamline the computational efficiency

Benchmarks:

To verify that our code is producing physically meaningful results, we benchmark our progress with the strong landau damping and two-stream instability. These problems have semi-analytic solutions for the growth and damping rates which we can use to validate our results. Here we display results for the two-stream instability.

$$f_0 = \frac{v^2}{\sqrt{2\pi}}\left[1 + \alpha\cos\left(\frac{x}{2}\right)\right]\exp\left(\frac{-v^2}{2}\right)$$

$$\alpha = 0.01, \quad (x, v) \in [0, 4\pi] \text{ x } [-10, 10]$$

Effective Resolution: $(256, 256)$

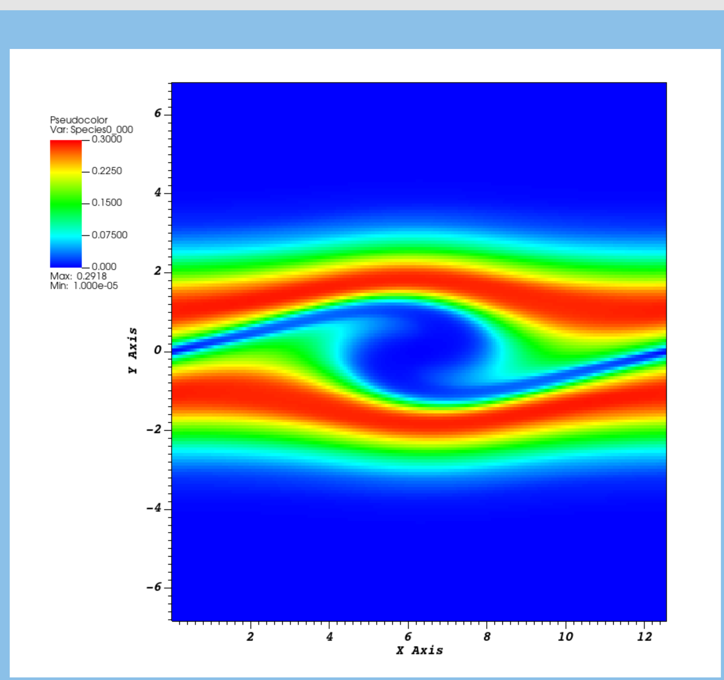

Figure 1: Snapshot of the two-stream instability taken at t=20.

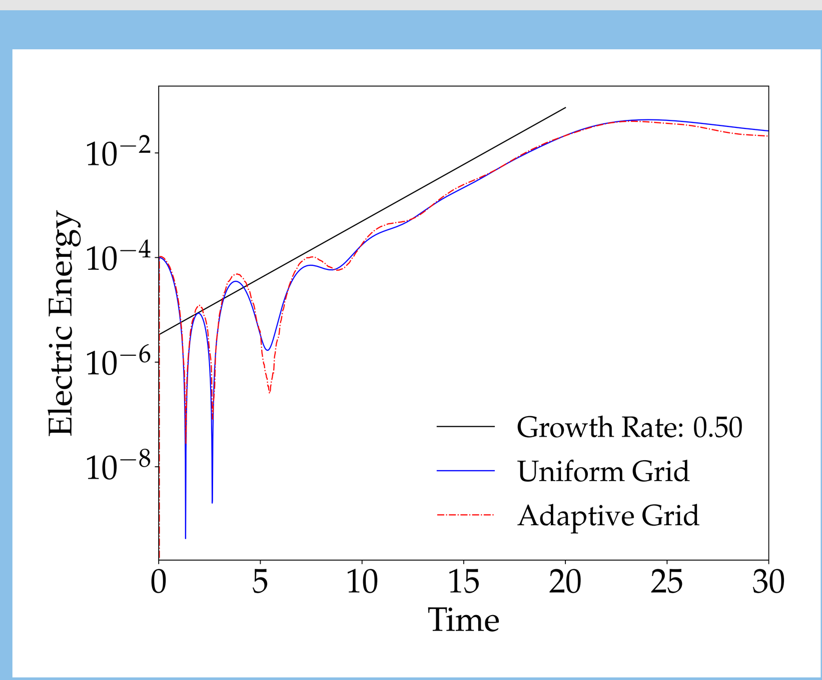

Figure 2: A comparison of the electric energy growth rate given in [4]. AMR run with 5 levels.

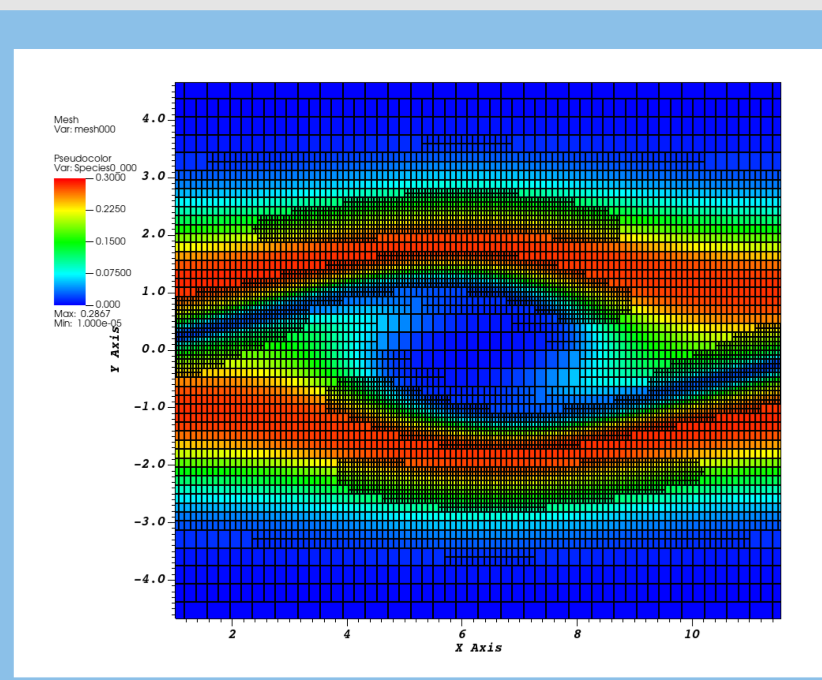

Figure 3: The same simulation taken with 7 levels and an effective resolution of (512, 512).

Plasma Wall interaction:

To build towards the Chodura sheath and modeling the plasma wall interaction, we begin with simulating an electrostatic plasma sheath [5, 6]. We employ an inflow boundary on the left and an absorbing wall on the right. For the Poisson equation, we use a floating-point boundary potential on each side. Our initial conditions are:

$$f_{e0} = \frac{n_{e0}}{\sqrt{2\pi}}\exp\left(\frac{-v^2}{2}\right), \quad f_{i0} = n_{i0}\delta(v - v_0)$$

Following [6], the ion velocity v_0 fully determines the steady state. Setting $v_0 = 0.2$, one obtains normalization $n_{i0} = 1.115\, n_{e0}$, wall potential $\phi_{wall} = -0.80$, and wall charge $Q_{wall} = -0.77$.

There are three ways to run *kobra*: uniform grid with Gauss-Seidel, uniform grid with FAS multigrid, adaptive grid with FAS multigrid. We compare the three modes of operation

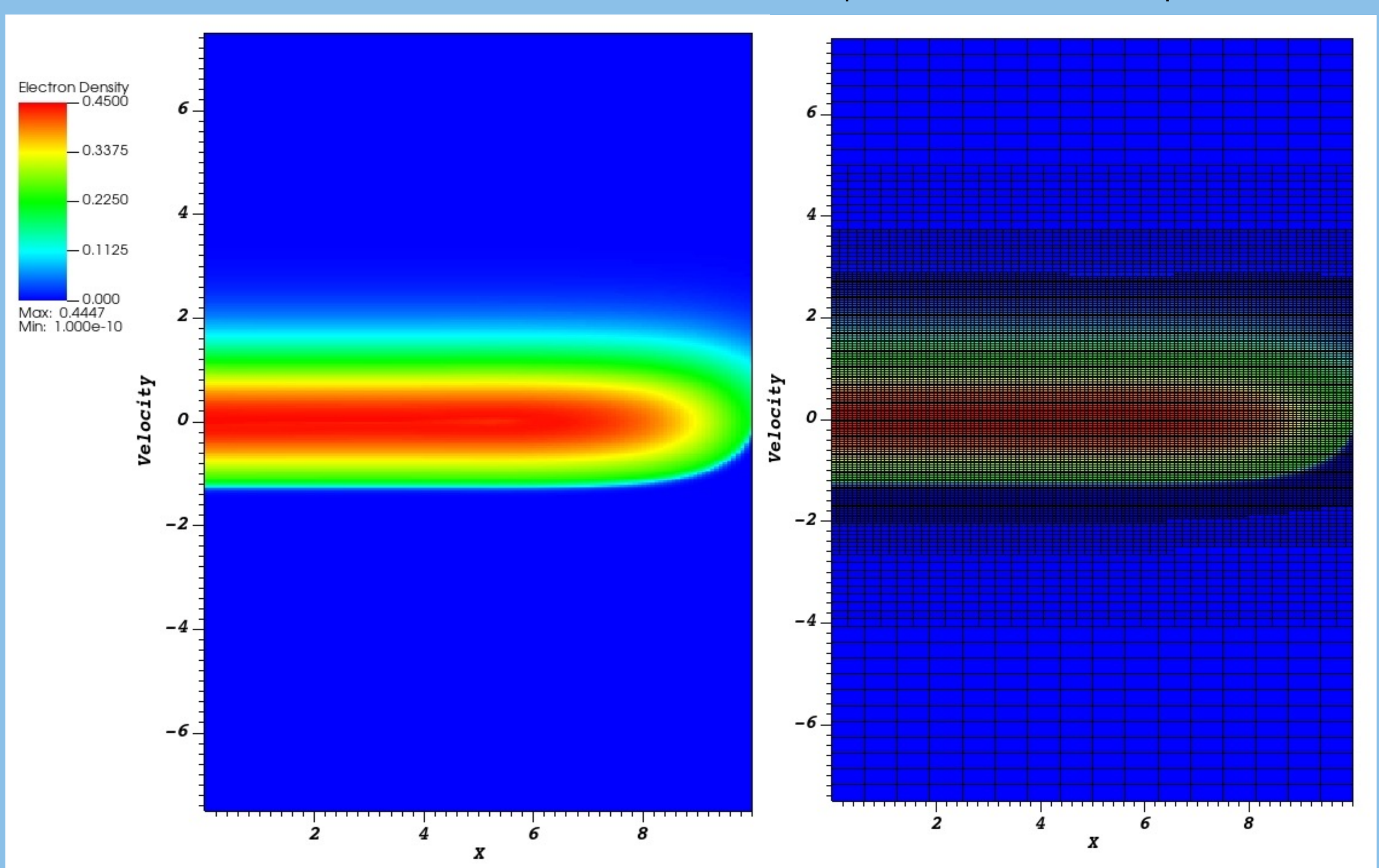

Figure 4: The converged electron plasma sheath (right) at t=500 with the overlayed adaptive grid (left). Simulations run with an effective resolution of $(x, v) = (128, 384)$. AMR run with 5 levels of refinement.

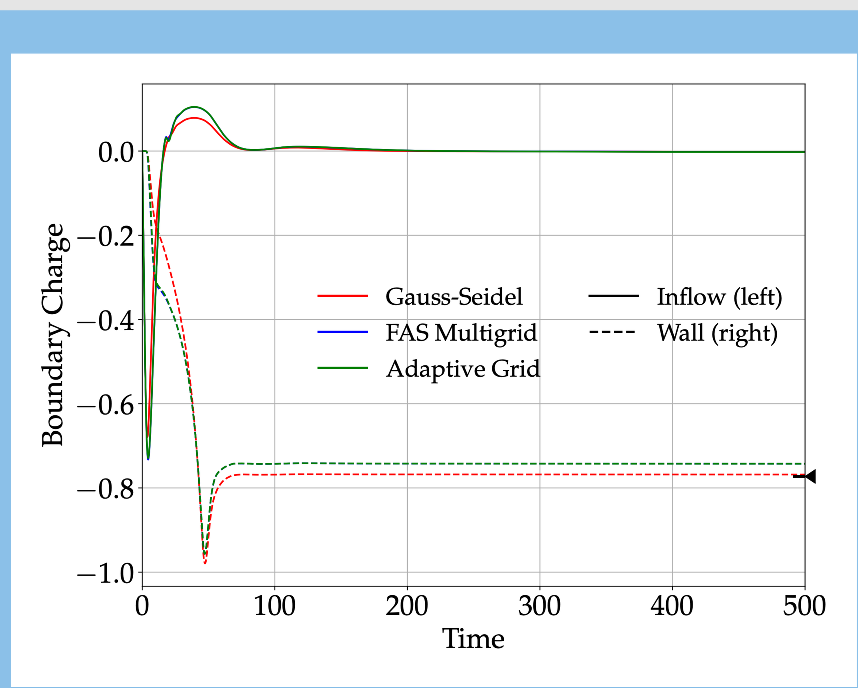

Figure 5: The time evolution of the charge at the boundaries. The left side connects to the quasi-neutral core and confirms our normalization n_{e0}. The right boundary yields Q_{wall}.

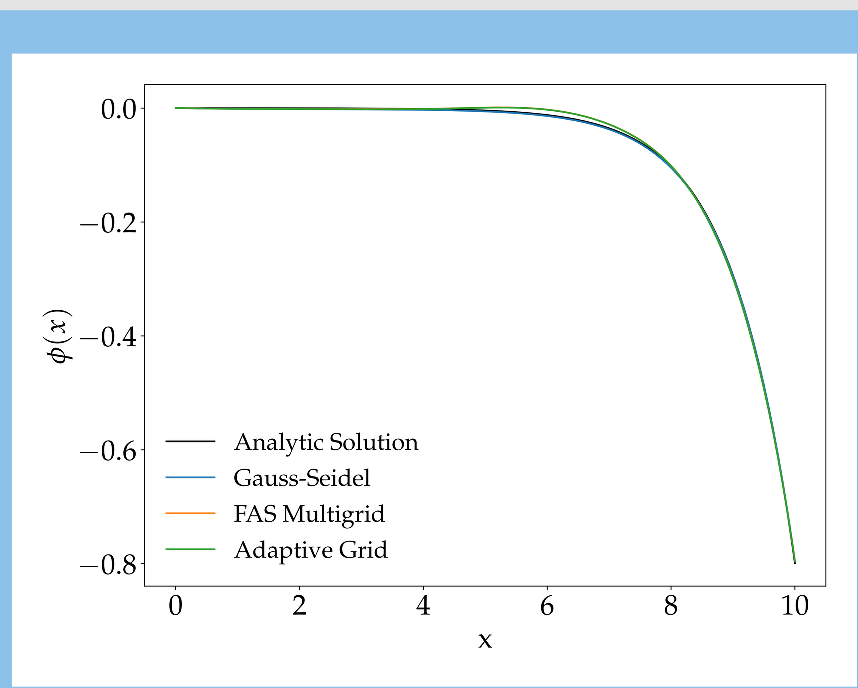

Figure 6: A comparison of the analytic potential distribution $\phi(x)$ with *kobra's* numeric results. The distribution is rescaled so the left boundary is 0. We reproduce the wall potential ϕ_{wall}.

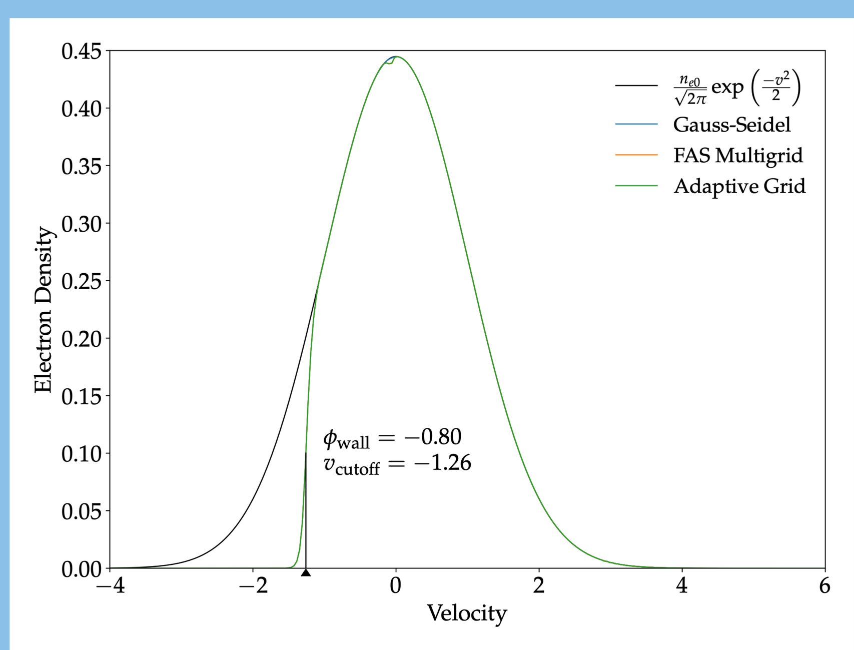

Figure 7: A slice of the electron profile taken at the core $(x = 0)$ against a reference Maxwellian distribution. Using ϕ_{wall}, we reproduce the analytic cutoff velocity $v_{cut} = -\sqrt{-2\phi_{wall}}$.

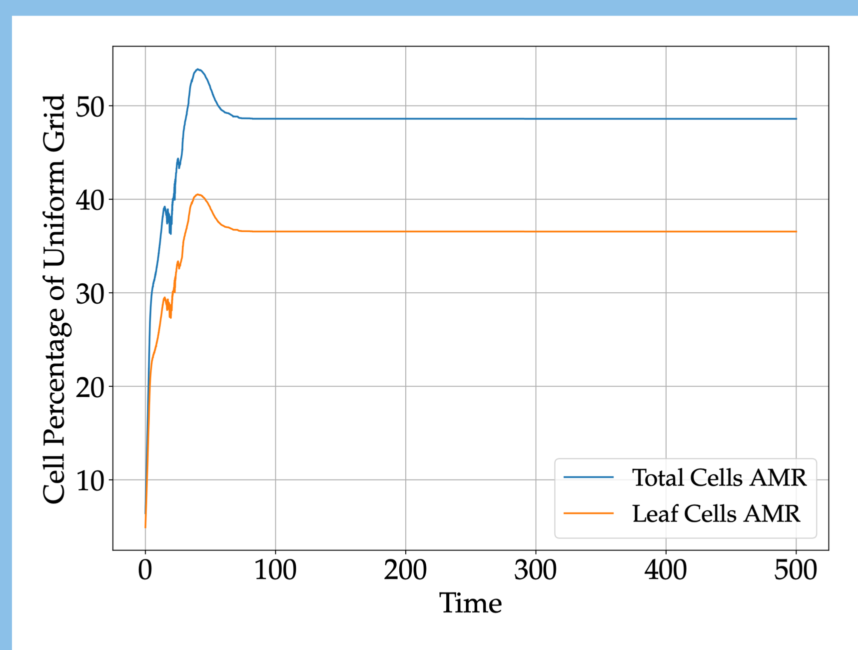

Figure 8: The number of cells in the AMR run expressed as a percentage of the total uniform grid cell count. We reproduce the analytic results with half of the uniform grid cells and half the wall time.

Conclusions:

We have a working Vlasov-Poisson code with an adaptive grid! We reproduce the damping and growth rates of standard kinetic benchmarks and plasma sheath physics. We demonstrate computational speeding using our AMR suite. We are ready to move to higher dimensional problems.

Next Steps:

- **Model the 1d3v "Chodura" sheath**
- **Anisotropic AMR/Differential timestep**
- **Promote to fully 4th order code**
- **Extend to Vlasov-Maxwell system**
- **Include additional physics**

Sources

[1] S. A. E. G. Falle. "A numerical scheme for multifluid magnetohydrodynamics". In: Monthly Notices of the Royal Astronomical Society 344.4 (Oct. 2003), DOI: 10.1046/j.1365-8711.2003.06908.x.
[2] J.A.F. Hittinger and J.W. Banks. "Block-structured adaptive mesh refinement algorithms for Vlasov simulation". In: Journal of Computational Physics 241 (2013) DOI: https://doi.org/10.1016/j.jcp.2013.01.030.
[3] Van Loo, Sven and Falle, S. A. E. G. and Hartquist, T. W. "Generation of density inhomogeneities by magnetohydrodynamic waves in two dimensions"
[4] Vogman, Genia, et al. "Fourth-Order Conservative Vlasov-Maxwell Solver for Cartesian and Cylindrical Phase Space Coordinates.", University of California, 2016.
[5] R. Chodura. "Plasma–wall transition in an oblique magnetic field". In: The Physics of Fluids 25.9 (Sept. 1982), ISSN: 0031-9171. DOI: 10.1063/1.863955.
[6] Schwager, L. A., and C. K. Birdsall. "Collector and source sheaths of a finite ion temperature plasma." *Physics of Fluids B: Plasma Physics*, vol. 2, no. 5, 1 May 1990, https://doi.org/10.1063/1.859279.

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