Determining distribution functions for one-dimensional force-free current sheet equilibria

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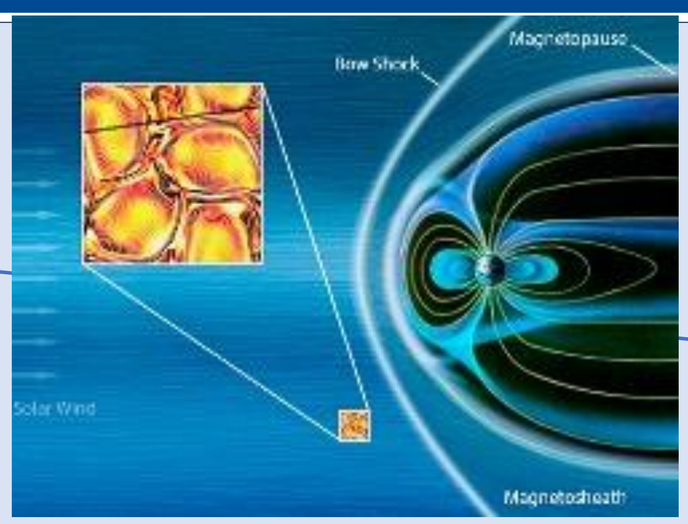
Current sheets ubiquitous in space plasmas

Often force-free

Here: collisionless plasmas

How does one determine self-consistent distribution functions for a given B field?



1D: only dependent on z (x, y invariant)

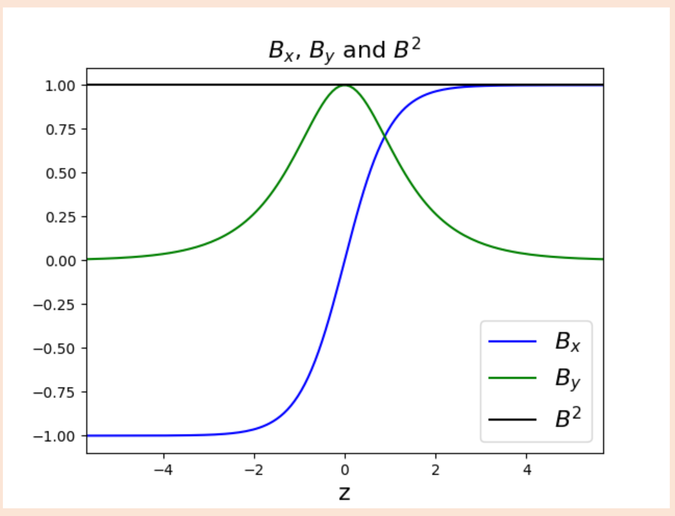
Magnetic field: $\boldsymbol{B} = [\mathrm{B_x}(\mathrm{z}), \mathrm{B_y}(\mathrm{z}), 0]$

Force-free: $\boldsymbol{j} \times \boldsymbol{B} = \boldsymbol{0}$

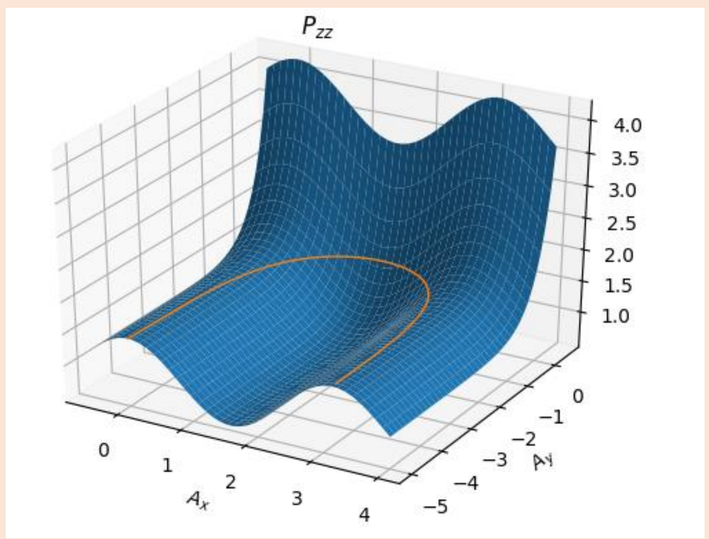
$\Longrightarrow \mathrm{B_x^2}(\mathrm{z}) + \mathrm{B_y^2}(\mathrm{z}) = \mathrm{B_T^2}$ =constant

$\mathrm{P_{zz}}(\mathrm{z})$ = constant

A Known Example: Force-free Harris Sheet with $B_y \to 0$ as $|z| \to \infty$



Given magnetic field
$\mathrm{B_y}(\mathrm{z}) \to 0$ as $|\mathrm{z}| \to \infty$!

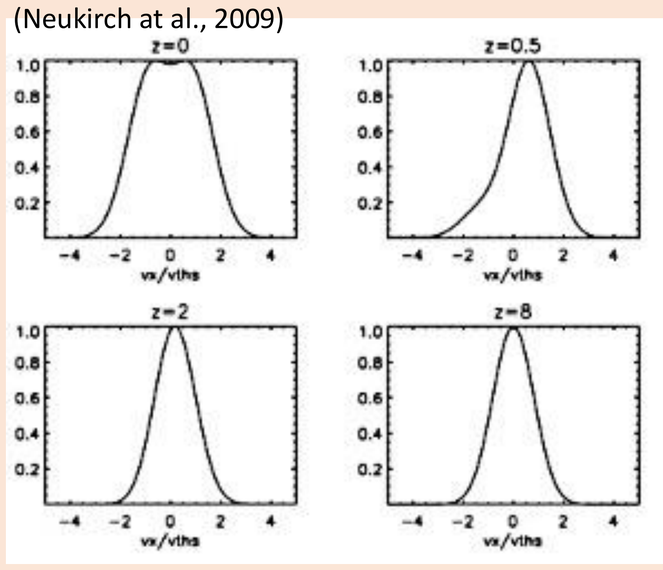


Calculate $\mathrm{A_x}(z), \mathrm{A_y}(z)$ (vector potential)

Determine $\mathrm{P_{zz}}(\mathrm{A_x}, \mathrm{A_y}) = \mathrm{P_1}(\mathrm{A_x}) + \mathrm{P_2}(\mathrm{A_y})$

Assume
$\mathrm{F}_s(H_s, p_{x,s}, p_{y,s}) \propto \exp(-\beta_s H_s)\, \mathrm{g_s}(p_{x,s}, p_{y,s})$

(Neukirch at al., 2009)



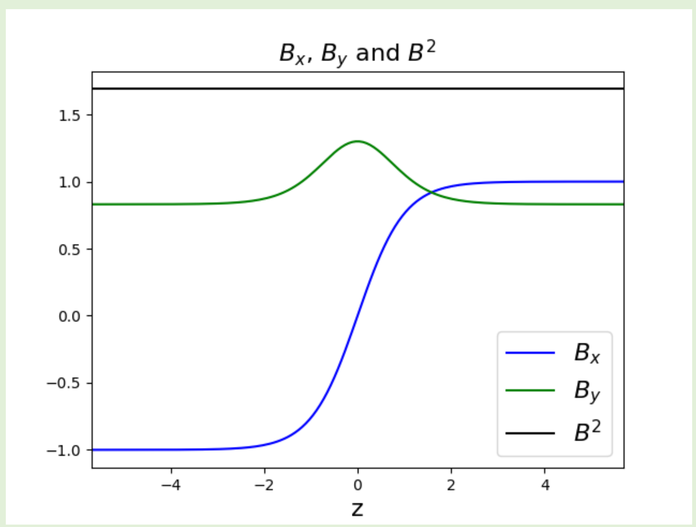
Solve integral equation for $\mathrm{g_s}$:

$$P_{zz}(A_x, A_y) = C \iint_{-\infty}^{\infty} \mathrm{K}(A_x, A_y; p_{x,s}, p_{y,s}) \mathrm{g_s}(p_{x,s}, p_{y.s})\, dp_{x.s} dp_{y.s}$$

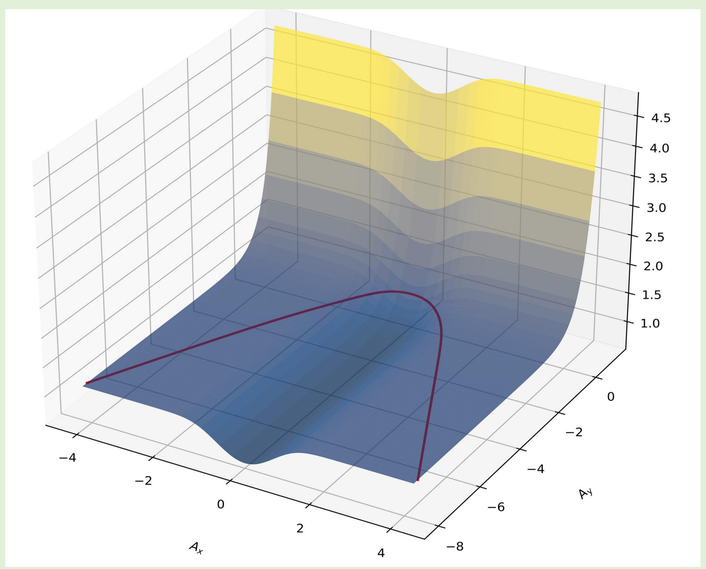
(Channell, 1976)

Obtain self-consistent DF: $F_s(H_s, p_{x,s}, p_{y,s})$

Problem: What if $B_y \to$ constant $\neq 0$ as $|z| \to \infty$?



B field as above, except
$\mathrm{B_y}(\mathrm{z}) \to \mathrm{const} \neq 0$ as $|\mathrm{z}| \to \infty$!



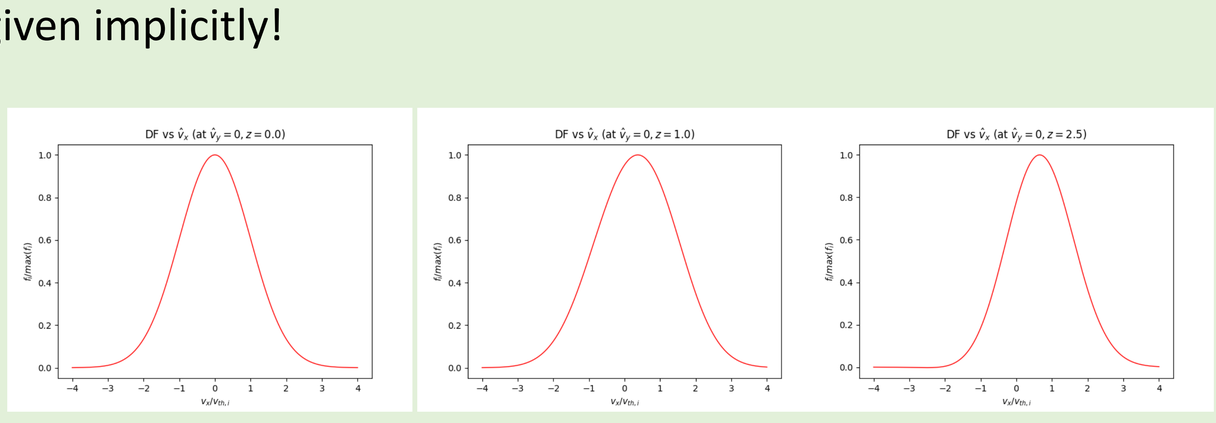
$\mathrm{P_1}(\mathrm{A_x})$ only given implicitly!

How to solve integral equation if LHS not known explicitly ?

Solution Method

Expand $\mathrm{p_{x,s}}$-dependent part of $\mathrm{g_s}$into a series of orthogonal functions (here Hermite functions)

Expansion coefficients: determined by Maclaurin series coefficients of $\mathrm{P_1}(\mathrm{A_x})$ (using implicit differentiation)



References:
Channell, P.J., PoF 19, 1541
Neukirch, T., Wilson, F., & Harrison, M.G., 2009, PoP 16, 122102