

# Energy-optimized scaling laws for self-guided laser wakefield accelerators

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Laser wakefield acceleration (LWFA) has evolved into a powerful electron acceleration scheme capable of sustaining accelerating fields that exceed those of conventional radio-frequency accelerators by several orders of magnitude [1]. In this method, an intense laser pulse propagating through a plasma excites a plasma wave, whose strong longitudinal electric fields can trap and accelerate electrons to relativistic energies. Over the past few decades, LWFA has attracted significant interest due to its potential for compact electron accelerators with applications such as novel radiation sources, muon production, radiotherapy, and investigations of strong-field quantum electrodynamics.

Efficient LWFA requires that the driving laser pulse maintains a high intensity over distances far exceeding its diffraction length. This necessitates some form of guiding within the plasma, which can be provided either externally or through self-guiding. Here, we concentrate on the latter. Self-guided propagation is appealing from an experimental standpoint because it avoids the need for precisely engineered guiding structures. On the other hand, self-guiding typically occurs in a strongly nonlinear regime, which can compromise stability and beam quality compared with externally guided configurations. These considerations make self-guided LWFA an attractive yet nontrivial candidate for systematic optimization.

Classical scaling relations for self-guided LWFA were established in the seminal work of W. Lu, *et al.* [2]. Building on these results, our aim in the present work is to derive scaling laws for the maximum electron energy and the corresponding acceleration distance in self-guided LWFA that (i) are expressed solely in terms of the laser energy and wavelength, (ii) yield the highest electron energy over the shortest acceleration distance possible, and (iii) are accompanied by the complete set of laser and plasma parameters required to enable the scaling. To accomplish our goal, we employ Bayesian optimization (BO), a machine-learning method well suited to optimizing objective functions that are expensive to evaluate. In our case, the objective function is the maximum electron energy, and its values are sampled using particle-in-cell (PIC) simulations performed in a quasi-three-dimensional geometry [3, 4] and in a Lorentz-

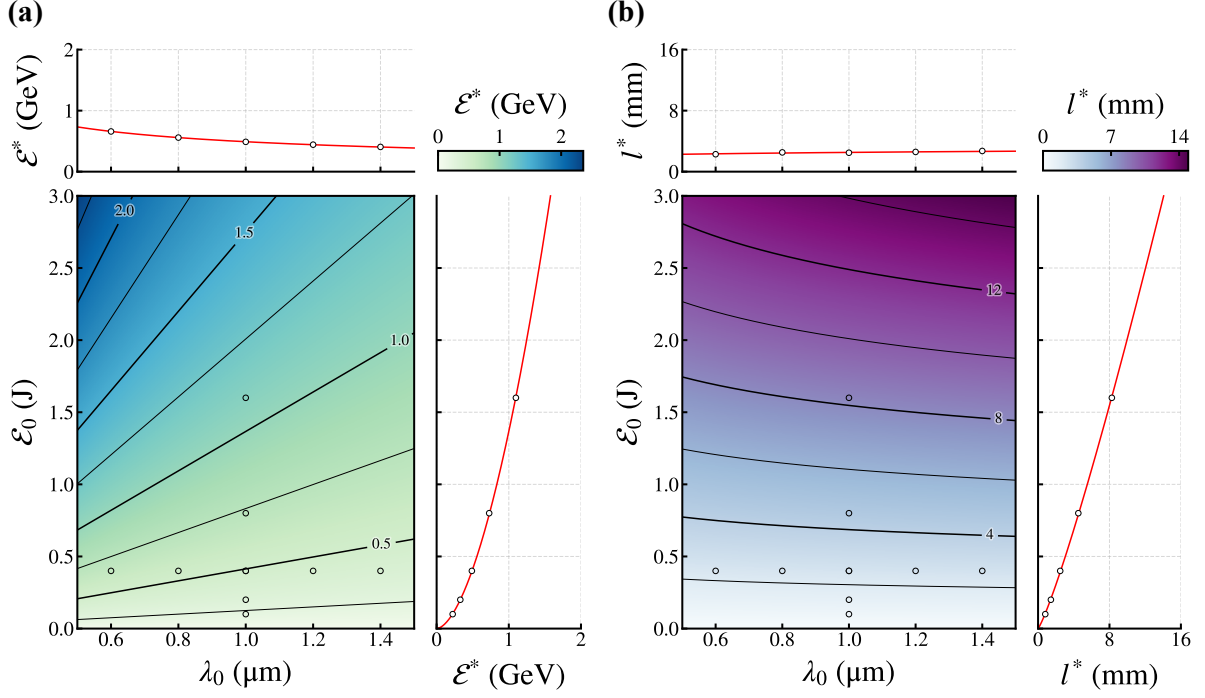


Figure 1: Energy-optimized scaling laws for (a) the maximum electron energy,  $\mathcal{E}^*$ , and (b) the corresponding shortest acceleration distance,  $l^*$ , in self-guided LWFA. In both panels, dots denote the optimal values extracted from the surrogate models, while solid red lines represent the analytical approximations of Eq. (1).

boosted frame [5, 6].

At the current stage, we concentrate on the maximum electron energy alone, leaving aside other beam characteristics (e.g., charge, energy spread, and divergence). This choice is motivated by applications where surpassing a certain energy threshold is the primary requirement, such as muon generation or nuclear activation. Furthermore, while the electron energy is determined mainly by the parameters of the laser and the background plasma, beam quality metrics are expected to depend strongly on a particular injection mechanism. Therefore, the energy-optimized regime identified here should remain applicable across various injection schemes, with beam-quality optimization addressed separately.

To reduce the dimensionality of the optimization problem, we impose several assumptions: (i) LWFA is driven by a linearly polarized Gaussian laser pulse with energy  $\mathcal{E}_0$ , wavelength  $\lambda_0$ , and a constant spectral phase; (ii) the plasma has a uniform density; (iii) the interaction operates in the matched regime [7, 8]; and (iv) electrons are externally injected and treated as test particles. These assumptions isolate the dominant physical dependencies while keeping the computational effort tractable.

As demonstrated in our recent study [9], under assumptions (i)–(iv) the LWFA problem can be fully described by only two dimensionless parameters:  $\mathcal{P}_0/\mathcal{P}_{\text{cr}}$  and  $\tau_0\omega_p$ , where  $\mathcal{P}_0$  and  $\tau_0$  are the power and duration (full width at half maximum of the intensity profile) of the laser pulse, respectively,  $\mathcal{P}_{\text{cr}}$  is the critical power for relativistic self-focusing [10], and  $\omega_p$  is the plasma frequency. In this work, we consider  $\mathcal{E}_0 \in [0.1, 1.6]$  J and  $\lambda_0 \in [0.6, 1.4]$   $\mu\text{m}$ , which covers the typical laser energies and wavelengths used in LWFA while also spanning a sufficiently large parameter region to allow extrapolation of the results.

For various laser energies and wavelengths selected from the above-mentioned intervals, we carry out BO and construct surrogate models for the maximum electron energy,  $\mathcal{E}_{e,\text{max}}$ , and the corresponding shortest acceleration length,  $l_{\text{acc}}$ , each based on 128 PIC simulations. By extracting and interpolating the optimal values of  $\mathcal{E}_{e,\text{max}}$  and  $l_{\text{acc}}$  (hereafter denoted by  $\mathcal{E}^*$  and  $l^*$ , respectively) from these models, we obtain new energy-optimized scaling laws for self-guided LWFA [11]:

$$\frac{\mathcal{E}^*}{m_e c^2} \approx 3.81 \left( \frac{\mathcal{E}_0}{\bar{\mathcal{E}}} \right)^{0.58} \quad \text{and} \quad \frac{l^*}{\lambda_0} \approx 0.75 \left( \frac{\mathcal{E}_0}{\bar{\mathcal{E}}} \right)^{0.85}, \quad (1)$$

where  $\bar{\mathcal{E}} = \lambda_0 m_e c^2 / r_e$ , with  $m_e$  denoting the electron mass,  $r_e$  the classical electron radius, and  $c$  the speed of light in vacuum. The optimal values extracted from the surrogate models, together with the approximations given by Eq. (1), are shown in Fig. 1.

The trends described by Eq. (1) are in reasonably good agreement with analytical models based on the linear quasi-static approximation, which predict  $\mathcal{E}^*/m_e c^2 \sim (\mathcal{E}_0/\bar{\mathcal{E}})^{2/3}$  and  $l^*/\lambda_0 \sim \mathcal{E}_0/\bar{\mathcal{E}}$  (see, e.g., Refs. [9, 12]). The remaining differences can be attributed to depletion of the driving laser energy in the plasma: at later stages of acceleration, the laser no longer has sufficient power to sustain efficient self-guiding, causing the acceleration to cease prematurely. Extrapolating Eq. (1) to, e.g., a 1 kJ, 1  $\mu\text{m}$  laser suggests electron energies exceeding 45 GeV over an acceleration length of  $\approx 1.9$  m. The corresponding input laser and plasma parameters can be readily obtained, see Eqs. (29)–(34) in Ref. [9].

In summary, we introduce a combined BO and PIC simulation framework that enables the derivation of generalized, energy-based scaling laws for self-guided LWFA driven by lasers of arbitrary energy and wavelength. These scalings are explicitly optimized to provide the maximum electron energy over the shortest acceleration distance and are accompanied by the complete set of input parameters required for their realization. The results obtained provide a practical guideline for designing state-of-the-art LWFA experiments operating at their fundamental performance limits.

## Acknowledgements

This work was supported by the NSF and Czech Science Foundation (NSF-GACR collaborative Grant No. 2206059 and Czech Science Foundation Grant No. 22-42963L). This work was supported by the Ministry of Education, Youth and Sports of the Czech Republic through the e-INFRA CZ (ID:90254).

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