Field-Aligned Flux Coordinates for Rectangular Magnetrons

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Magnetron sputtering is a central enabling technology for plasma surface engineering and thin film deposition. Over the years, a broad spectrum of technological variants has emerged, ranging from conventional DC and RF sputtering to more advanced process concepts such as High Power Impulse Magnetron Sputtering (HiPIMS). The increasing complexity of modern magnetron sputtering discharges has substantially intensified the need for predictive theoretical models capable of describing electron confinement, transport processes, ionization dynamics, and plasma self-organization. As a consequence, theoretical modeling has become an increasingly important component of magnetron plasma research.

In this context, axisymmetric magnetrons have traditionally served as the preferred reference configuration for theoretical investigations. Their symmetry provides the basis for a variety of powerful mathematical concepts ranging from simplified transport descriptions to global field-aligned coordinate systems. Industrial applications, however, rely almost exclusively on large rectangular devices designed for high-throughput coating. From a theoretical perspective, these systems are substantially more difficult to model because the mathematical structures associated with axial symmetry are no longer available.

A particularly important example is given by field-aligned "Clebsch" coordinate systems. Their basic idea consists of representing the magnetic field by scalar functions which remain constant along magnetic field lines and thus provide a coordinate system intrinsically adapted to the magnetic geometry. For the description of partially magnetized plasmas, such coordinates provide a natural framework for the formulation of transport equations and kinetic models. In axisymmetric magnetrons, their construction is greatly facilitated by symmetry and can be based on well-established mathematical concepts. Previous work has shown that this naturally leads to a global coordinate system covering the entire magnetically confined plasma region [1]. In the present work, we address the considerably more difficult question whether an analogous construction remains possible for realistic rectangular magnetron geometries [2].

Field-aligned Clebsch coordinates are based on two scalar functions, the flux function $\Psi(\boldsymbol{r})$ and the generalized azimuth $\Theta(\boldsymbol{r})$, which represent the magnetic field in the form

$$\boldsymbol{B} = \nabla\Psi \times \nabla\Theta. \tag{1}$$

The function Ψ labels magnetic flux surfaces, while the pair (Ψ, Θ) uniquely labels individual magnetic field lines. Together with the arc-length coordinate S along field lines, this defines a complete field-aligned coordinate system. A particularly useful property is that the Jacobian of the transformation is directly given by the magnetic field strength:

$$J = \frac{\partial(\Psi, \Theta, S)}{\partial(x, y, z)} = (\nabla\Psi \times \nabla\Theta) \cdot \nabla S = \boldsymbol{B} \cdot \nabla S = B. \tag{2}$$

For axisymmetric magnetrons, the construction of such coordinates is straightforward. The flux coordinate Ψ is provided by the azimuthal component of the vector potential, $\Psi = rA_\theta$, while the second coordinate is simply the geometric azimuth $\Theta = \theta$. The familiar magnetic dome then defines a globally regular domain whose field lines possess the characteristic arc-like structure of magnetron discharges.

Plasma modeling, however, does not require the entire magnetic dome, but only the subset of active field lines along which electrons remain sufficiently magnetized throughout their motion. The question therefore arises whether an analogous family of active field lines, together with the corresponding field-aligned coordinates, can also be constructed for rectangular magnetrons.

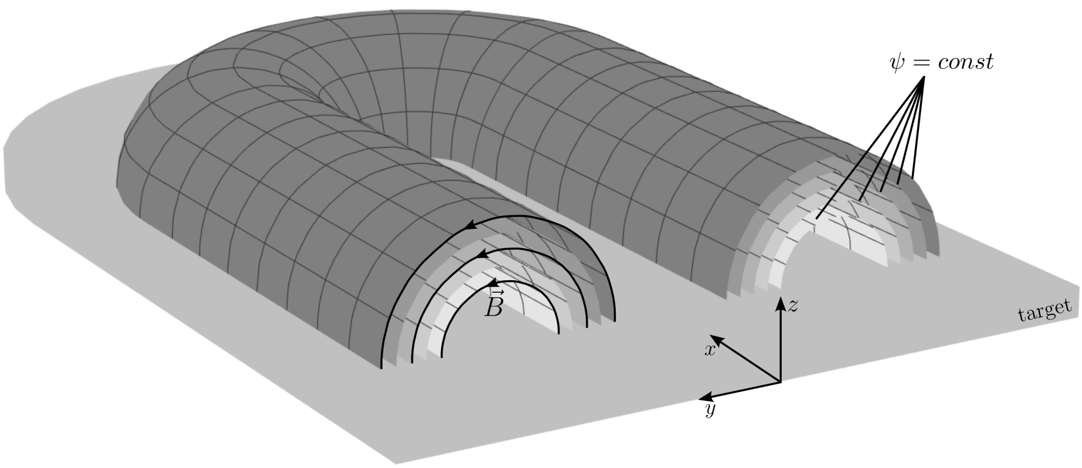

Figure 1: Idealized rectangular magnetron geometry illustrating the concept of field-aligned Clebsch coordinates in the magnetized plasma region. The shaded surfaces represent a family of flux surfaces $\Psi = \mathrm{const}$, while the embedded mesh indicates the field-aligned coordinates.

We consider a planar magnetron whose cathode has the shape of an elongated stadium. The centerline of this domain defines the characteristic racetrack curve of the magnetron. Starting from the axisymmetric case, one might imagine constructing the corresponding rectangular field by separating a circular magnetron into two halves and inserting straight sections while preserving the original field structure. However, this intuitive "slice-and-insert" construction would not satisfy the magnetostatic vacuum conditions.

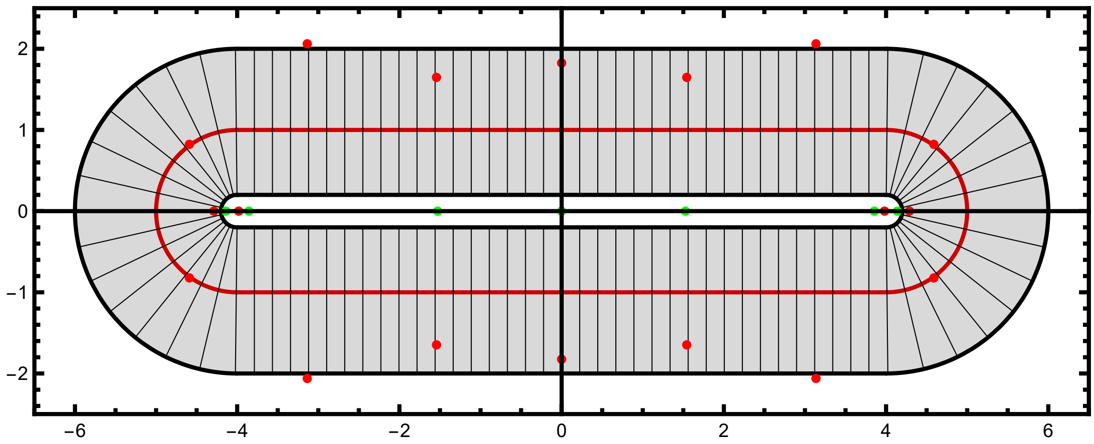

Figure 2: Stadium geometry in the cathode plane $z = 0$ together with the associated coordinate system serving as the scaffold for the field construction. The red curve indicates the prescribed racetrack where the normal field vanishes. Red and green markers denote the positions and polarities of an optimized ensemble of discrete magnetic dipoles located beneath the cathode.

To construct a physically realizable magnetic field, we represent the magnetic sources by an ensemble of point dipoles located beneath the cathode. Their positions and strengths are optimized so as to minimize deviations from the prescribed racetrack field while simultaneously enforcing a central magnetic null. The resulting configuration is shown in Figure 2.

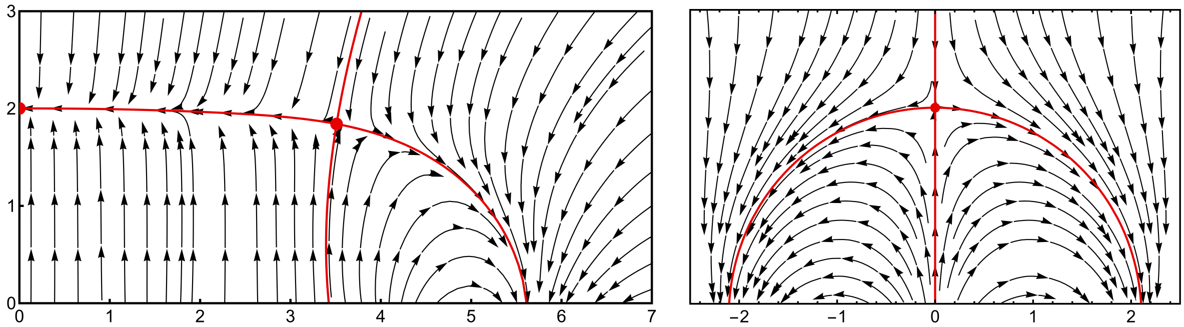

Figure 3: Side views of the optimized magnetic field. Field lines are in black, separatrices in red. The prescribed central null is accompanied by two additional off-axis nulls. As a consequence, the familiar magnetic dome of the axisymmetric case disappears.

Owing to the bifurcation of the central magnetic null, the optimized magnetic field no longer exhibits the dome structure known from axisymmetric magnetrons. In spite of this, however, a sufficiently large regular domain $\mathcal{R}$ still exists in which magnetic field lines remain arc-like. A subset of $\mathcal{R}$, the active region $\mathcal{A}$, is of even more importance: Electrons remain magnetized only where the magnetic field remains sufficiently strong. This physically relevant domain forms a deformed half-toroidal structure.

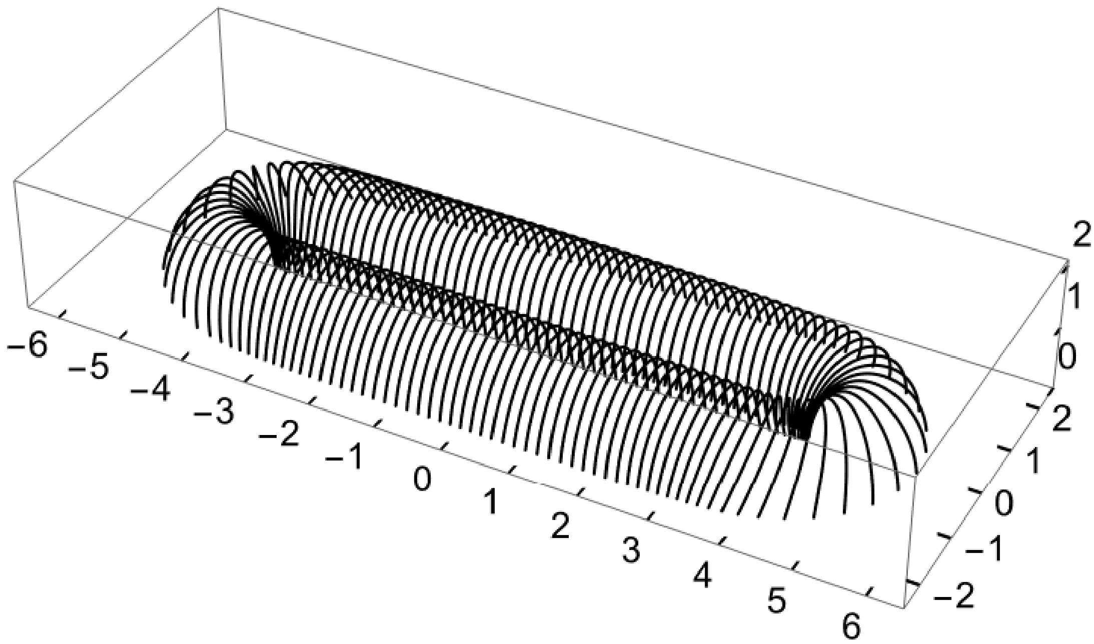

Figure 4: Field lines connected to the last active neutral curve. Enclosed is the active region $\mathcal{A}$, the magnetized domain between the cathode and the associated flux surface $\Psi(x, y, z) = \mathrm{const}$. The ensemble of 128 field lines visually reconstructs the corresponding flux surface.

The construction of the Clebsch coordinates itself is an exercise in advanced vector analysis. First, a family of neutral curves in the planes $z = \mathrm{const}$ is defined, satisfying $B_z(x, y, z) = 0$. These curves can be labeled by the flux Ψ. The regularity condition permits the introduction of a generalized azimuth coordinate Θ. Both functions are then extended along magnetic field lines. A non-trivial mathematical analysis shows that the resulting fields $\Psi(\mathbf{r})$ and $\Theta(\mathbf{r})$ indeed satisfy the Clebsch representation (details in Ref. [2]). Since the active region $\mathcal{A}$ lies entirely inside the regular domain $\mathcal{R}$, the coordinate construction remains possible exactly where it is needed.

References

[1] R.P. Brinkmann and D. Krüger, *Phys. Plasmas* **27**, 053504 (2020)

[2] R.P. Brinkmann, D. Krüger, J. Kallähn, K. Köhn, L. Vogelhuber, Y. Sharova, L. Xu, and D. Eremin, *Phys. Plasmas* **33**, 063508 (2026)