ESR2D: a two-dimensional Fourier-space global gyrokinetic eigenvalue code for the ion-temperature-gradient modes in tokamaks

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Introduction

The ion-temperature-gradient (ITG) mode is one of the most extensively studied drift-wave instabilities [1]. In the linear regime, unlike initial-value codes which require time evolution, eigenvalue codes can efficiently compute eigenvalues and eigenmode structures, which are useful for analyzing experimental observations [2]. Existing eigenvalue solvers including solvers developed in ballooning space and Fourier-space rely on either high toroidal number approximations or simplifying assumptions (e.g., well-circulating particles). In this work, we derive the two-dimensional (2D) gyrokinetic eigenvalue equations for the ITG modes in the poloidal Fourier space based on the Vlasov-Poisson system, and develop the ESR2D (2D eigenvalue solver in real space) code. This code treats passing and trapped ions in a unified way without simplifying assumptions, and it applies to arbitrary wavelengths. In the linear ITG Cyclone test with adiabatic electrons, the ESR2D code benchmarks well against the gyrokinetic initial-value codes GENE, NLT and ORB5. It is found that two toroidal branches of ITG modes coexist in the system. ESR2D is also applied to solving ITG modes in weak magnetic shear configurations near the magnetic axis, and the results agree well with NLT.

Basic equations in ESR2D

In this section, we briefly introduce the basic model equations including the Fourier-space gyrokinetic Vlasov equation and field equation in ESR2D.

We apply time domain Laplace transformation and Fourier transformations in the toroidal angle ζ and the poloidal angle χ to the perturbation of the physical quantity $Q(r,\chi,\zeta,t)$:

$$Q(r,\chi,\zeta,t) = \mathrm{e}^{-\mathrm{i}(n\zeta+\omega t)} \sum_m \mathrm{e}^{\mathrm{i}m\chi} \bar{Q}_{n,m}(r), \tag{1}$$

$$\bar{Q}_{n,m}(r) = \hat{Q}_n(z,m), \tag{2}$$

with r the radial coordinate, n the toroidal mode number, m the poloidal mode number, and $z \equiv nq(r) - m$. Here, Q can be either the electrostatic perturbation $\delta\phi$ or the perturbed distribution function δf. In a toroidal axisymmetric tokamak, the equations of different toroidal mode number n are independent, and the subscript n will be omitted in the following for simplicity.

Applying the transformation equations (1) and (2) to gyrokinetic Vlasov equation [3], with a decomposition of the perturbed distribution function of ions δf_i in terms of its adiabatic and non-adiabatic components, $\delta f_i = -\frac{e_i F_{0i}}{T_i}\langle\delta\phi\rangle_{\text{gy}} + g_i$, we obtain the Vlasov equation in the Fourier space, which is written as

$$\sum_p(\omega_p^0+\omega_p^z\partial_z+a_p\partial_{v_\parallel})\hat{g}_i(z-p,m+p,v_\parallel,\mu) = -\frac{e_iF_{Mi}^{(0)}}{T_i}\sum_p\omega_p^t\langle\delta\hat{\phi}\rangle_{\text{gy}}(z-p,m+p). \tag{3}$$

with $v_\parallel$ the parallel velocity, μ the magnetic moment, and e_i the ion charge. Here, the equilibrium distribution function is taken as the local Maxwellian distribution, and the gyro-average operator is defined as $\langle Q\rangle_{\text{gy}} \equiv \frac{1}{2\pi}\oint Q(\xi)d\xi$, with ξ the gyro-angle.

The quasi-neutrality equation with adiabatic electrons in the Fourier space is

$$-(\frac{e_i^2n_i}{T_i}+\frac{e_e^2n_e}{T_e})\delta\hat{\phi}(z,m)+e_i\frac{2\pi B^{(0)}}{m_i}\int_{-\infty}^{+\infty}\mathrm{d}v_\parallel\int_0^{+\infty}\mathrm{d}\mu\sum_p\sigma_p\left\langle\hat{g}_i\right\rangle_{\text{gy}}(z-p,m+p,v_\parallel,\mu)=0. \tag{4}$$

Here, ω_p^0, ω_p^z, a_p,ω_p^t, and σ_p are Fourier coefficients, m_i is the ion mass, e_e is the electron charge, T_i and T_e are the equilibrium temperatures of ions and electrons, respectively, n_i and n_e are the equilibrium densities of ions and electrons, respectively. Equations (3) and (4) construct the 2D eigenvalue problem in the Fourier space. The boundary conditions are $\hat{g}_i(z,m,v_\parallel,\mu)|_{z,m,v_\parallel\to\pm\infty} = 0$ and $\delta\hat{\phi}(z,m)|_{z,m\to\pm\infty} = 0$.

After discretization, the Vlasov equation (3) becomes

$$[\mathbf{A}(\omega)]\,[\hat{\mathbf{g}}] = [\mathbf{B}(\omega)]\,[\delta\hat{\phi}], \tag{5}$$

Here, $[\hat{\mathbf{g}}]$ and $[\delta\hat{\phi}]$ represent the discretized column vectors of the perturbed distribution function and perturbed electrostatic potential, respectively. Solving $[\mathbf{C}(\omega)] \equiv [\mathbf{A}(\omega)]^{-1}[\mathbf{B}(\omega)]$ with the sparse linear solver PARDISO, and substituting $[\hat{\mathbf{g}}]$ into the quasi-neutrality equation (4), we obtain the algebraic eigenvalue equation, which is written as

$$[\mathbf{M}(\omega)]\,[\delta\hat{\phi}] = 0. \tag{6}$$

The ESR2D code employs Newton's method to solve the algebraic eigenvalue equation (6) for eigenvalue ω.

Numerical results

In this section, we present two benchmark tests: solving ITG modes with Cyclone parameters, and solving ITG modes in weak magnetic shear configurations near the magnetic axis.

The equilibrium set-up of the Cyclone benchmark is chosen the same as that in Ref. [4]. We scan over toroidal mode number n to solve the 2D eigenvalue problem of the ITG modes with adiabatic electrons. Figure 1 shows the eigenvalues from ESR2D and those from the initial-value codes GENE, NLT and ORB5. The ESR2D code obtains two branches of ITG modes (labeled mode 1 and mode 2). The results from GENE are consistent with those of the first branch, while the results from NLT are consistent with those of the most unstable branch, with both the real frequencies and the growth rates obtained by the NLT code jumping from the first branch to the second branch near $n = 35$. The ORB5 results agree well with the first branch at low-n($n < 25$), but deviate at high-n($n \geq 25$), likely due to the long-wavelength model adopted in ORB5.

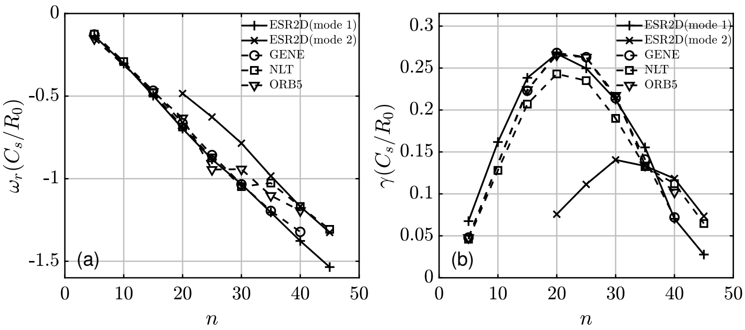

Figure 1: Real frequencies ω_r (a) and growth rates γ (b) versus the toroidal mode number n, obtained by ESR2D, GENE, NLT and ORB5, normalized by C_s/R_0 with $C_s = \sqrt{T_i(r_0)/m_i}$.

For the weak magnetic shear benchmark, the equilibrium configuration is the same as that in reference [5].

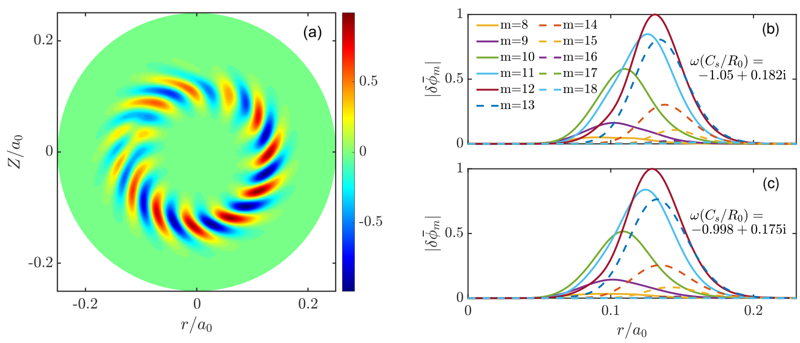

Figure 2: Eigenmode structures and eigenvalues of ITG modes in weak magnetic shear configuration near the magnetic axis ($n = 10$). (a): 2D mode structures; (b)-(c): radial structures and eigenvalues obtained by ESR2D and NLT, respectively.

Figure 2 shows the eigenmode structures and eigenvalues of ITG modes at toroidal mode number $n = 10$ obtained by ESR2D and NLT. Good agreement between ESR2D and NLT is observed.

Conclusions

In this work, we have developed the ESR2D code to solve the 2D gyrokinetic eigenvalue problem for the ITG modes in tokamaks. The code treats passing and trapped ions in a unified way without simplifying assumptions. Benchmark results against GENE, NLT, and ORB5 show good agreement for Cyclone parameters and we find that two toroidal branches of ITG modes coexist in the system by using the ESR2D code. ESR2D is also capable of solving for ITG modes in weak magnetic shear configurations near the magnetic axis, and the results agree well with those obtained by NLT.

Acknowledgments

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