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Drifting cold-ion flow boundary condition at the magnetic presheath entrance

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INTRODUCTION AND MOTIVATION

- Inclusion of drifts in wall boundary conditions is necessary because grazing magnetic incidence attenuates ion parallel transport to the wall
- Drifts have been treated as explicit corrections and assumed small [1,2]
- Implementations inconsistent between different fluid codes [3]

ASSUMPTIONS AND ORDERINGS

- Wall at $x = 0$, plasma at $x > 0$, magnetic field $\vec{B} = B(-\vec{e}_x \sin \alpha + \vec{e}_z \cos \alpha)$ is constant and electric field is $\vec{E} = -\vec{e}_x \partial \phi / \partial x - \vec{e}_y \partial \phi / \partial y - \vec{e}_z \partial \phi / \partial z$
- Quasineutrality + adiabatic electrons with $T_e(y)$ and $\bar{n}(y)$ inputs:

$$n_i(x, y) = n_e(x, y) = \bar{n}(y) \exp\left(\frac{e\phi(x, y)}{T_e(y)}\right) \equiv n(x, y)$$
- Cold ions $T_i = 0 \implies \nabla p_i = 0$
- Order unity fluctuations $\Delta \phi \sim T_e/e$ and $\Delta n \sim n$
- Long wavelength in y direction: $\partial_y \phi / B \sim \delta c_S$ with $c_S = \sqrt{Z T_e / m_i}$ and $\delta \ll 1$
- Drifts $\perp$ wall compete with parallel streaming: $u_{\parallel} \sin \alpha \sim \alpha c_S$ with $\alpha \sim \delta \ll 1$

PARTICLE AND MOMENTUM EQUATIONS

$$\begin{aligned} \frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(n u_x) + \frac{\partial}{\partial y}(n u_y) &= S_n \\ \frac{\partial u_{\parallel}}{\partial t} + u_x \frac{\partial u_{\parallel}}{\partial x} + u_y \frac{\partial u_{\parallel}}{\partial y} - \frac{\Omega}{B} \sin \alpha \frac{\partial \phi}{\partial x} &= S_{u_{\parallel}} \quad \Omega = ZeB/m_i \\ u_x &= -u_{\parallel} \sin \alpha - \frac{1}{B} \frac{\partial \phi}{\partial y} \cos \alpha, \quad u_y = \frac{1}{B} \frac{\partial \phi}{\partial x} \cos \alpha, \quad n = \bar{n}(y) \exp\left(\frac{e\phi(x, y)}{T_e(y)}\right) \end{aligned}$$

$$\begin{aligned} \frac{\partial \ln n}{\partial t} - \left[u_{\parallel} \tan \alpha + \frac{1}{B} \frac{\partial \phi}{\partial y} - \frac{c_S^2}{\Omega} \frac{\partial \ln n}{\partial y} \right] \frac{\partial \ln n}{\partial x} \cos \alpha - \sin \alpha \frac{\partial u_{\parallel}}{\partial x} &= S_{\ln n} \\ \frac{\partial u_{\parallel}}{\partial t} - \left[u_{\parallel} \tan \alpha + \frac{\partial \phi}{\partial y} \right] \frac{\partial u_{\parallel}}{\partial x} \cos \alpha - c_S^2 \left[\tan \alpha - \frac{1}{\Omega} \frac{\partial u_{\parallel}}{\partial y} \right] \frac{\partial \ln n}{\partial x} \cos \alpha &= S_{u_{\parallel}} \end{aligned}$$

$$\begin{aligned} \left[\frac{\partial}{\partial t} + (-u_{\parallel}^* \pm c_S^*) \sin \alpha \frac{\partial}{\partial x} \right] (U_{\mp} \mp u_{\parallel}) &= \left(c_S^* \mp \frac{1}{2} \frac{c_S^2}{\sin \alpha} \frac{\partial \ln n}{\partial y} \right) S_{\ln n} \mp S_{u_{\parallel}} \\ dU_{\mp} &= \left(c_S^* \mp \frac{1}{2} \frac{c_S^2}{\sin \alpha} \frac{\partial \ln n}{\partial y} \right) d \ln n \\ u_{\parallel}^* &= u_{\parallel} + \frac{1}{B \tan \alpha} \frac{\partial \phi}{\partial y} - \frac{c_S^2}{2 \tan \alpha} \frac{\partial \ln n}{\partial y} \\ c_S^* &= c_S \sqrt{1 - \frac{1}{\Omega \tan \alpha} \frac{\partial u_{\parallel}}{\partial y} + \frac{c_S^2}{4 \Omega^2 \tan^2 \alpha} \left(\frac{\partial \ln n}{\partial y} \right)^2} \end{aligned}$$

BOHM-CHODURA CONDITION = NOZZLE CONDITION: NO INFORMATION PROPAGATES FROM THE WALL

Characteristics move at speed $\left(\frac{dx}{dt} \right)_{\pm} = (-u_{\parallel}^* \pm c_S^*) \sin \alpha$

At wall $x=0$ $\left(\frac{dx}{dt} \right)_{+} = 0 \Rightarrow u_{\parallel}^* = c_S^*$ (effective sound speed)

PERTURBATIVE APPROXIMATION TO THE BOHM CONDITION

$$\delta/\alpha \ll 1 \Rightarrow u_{\parallel} \simeq u_{\parallel, \text{pert}} \equiv c_S - \frac{1}{\tan \alpha} \left(\frac{1}{B} \frac{\partial \phi}{\partial y} - \frac{1}{2} \frac{c_S^2}{\Omega} \frac{\partial \ln n}{\partial y} + \frac{1}{4} \frac{c_S^2}{\Omega} \frac{1}{T_e} \frac{\partial T_e}{\partial y} \right)$$

Sometimes this is simplified to $u_{\parallel} \approx c_S - \frac{1}{B \tan \alpha} \frac{\partial \phi}{\partial y}$

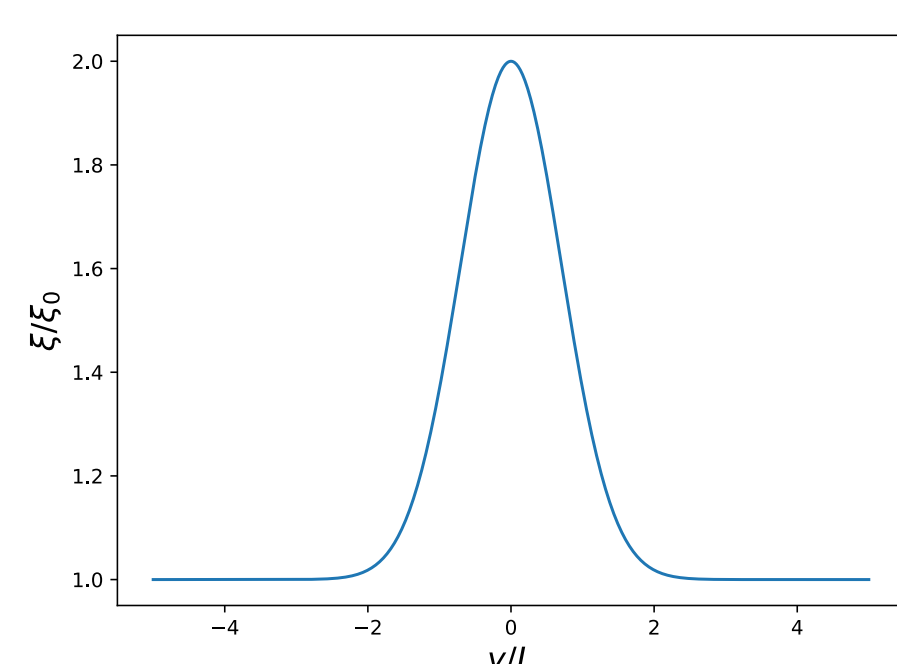
TOY GAUSSIAN MODEL OF A FLUCTUATION REACHING THE TARGET

$$\xi(y) = \xi_0 \left(1 + \exp(-y^2/L^2) \right)$$

$$L = \rho_S / \delta \text{ with } \rho_S = c_S / \Omega$$

$$\xi \in \{\phi|_{x=0}, T_e, \bar{n}\}$$

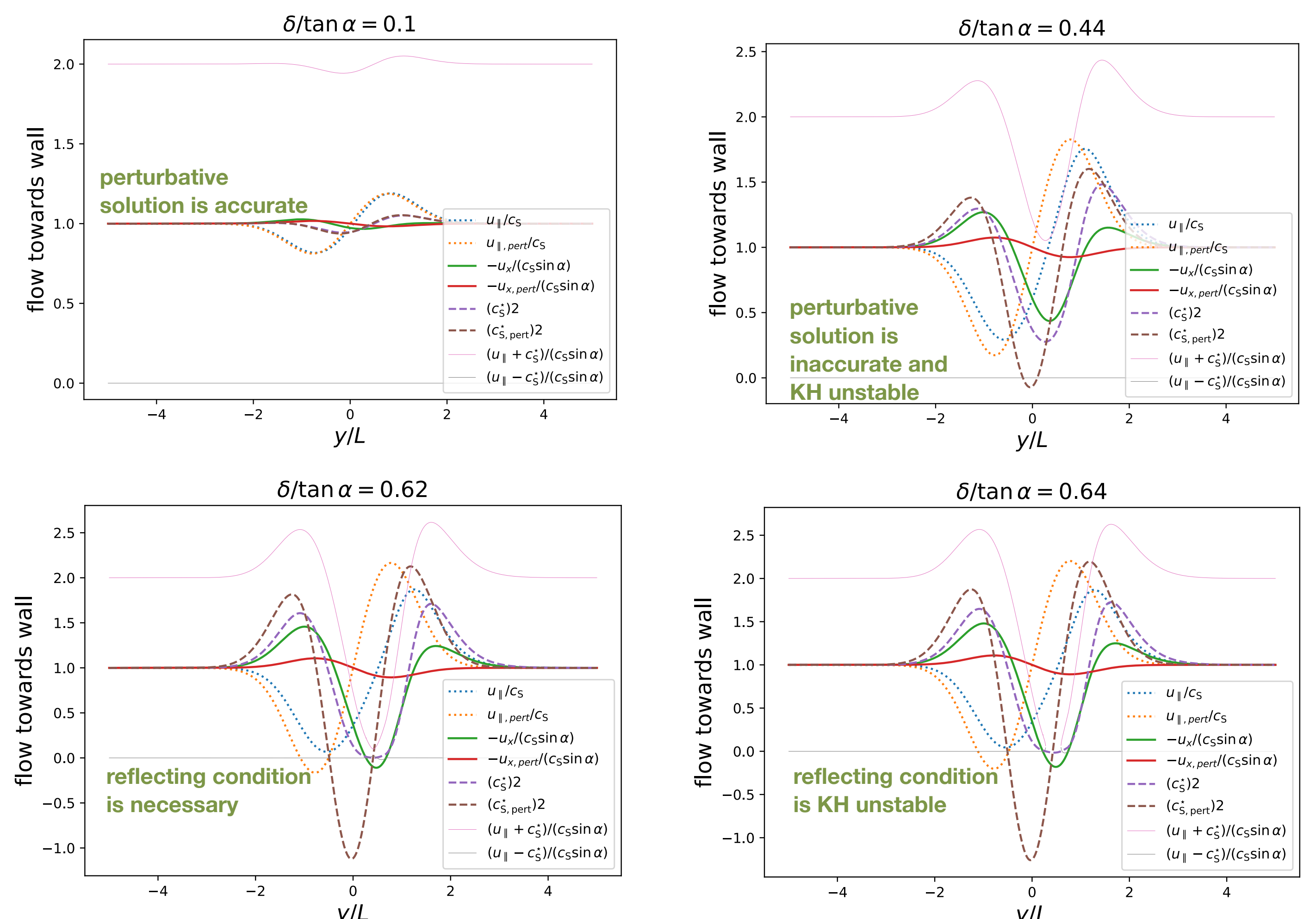
$$e\phi_0 / T_{e,0} = 3$$



SOLVING THE FULL BOUNDARY CONDITION

- Integrate in +ve y direction from $y_{\text{left}}/L = -5$ (where $\partial/\partial y \approx 0$, $u_{\parallel} \simeq u_{\parallel, \text{pert}}$)

$$\frac{c_S^2}{\Omega \tan \alpha} \frac{\partial u_{\parallel}}{\partial y} = \left(u_{\parallel} + \frac{1}{B \tan \alpha} \frac{\partial \phi}{\partial y} - \frac{c_S^2}{2 \tan \alpha} \frac{\partial \ln n}{\partial y} \right)^2 - c_S^2 - \frac{c_S^4}{4 \Omega^2 \tan^2 \alpha} \left(\frac{\partial \ln n}{\partial y} \right)^2$$
- With toy model, solution only depends on $\delta/\tan \alpha$
- Plot parallel flow $u_{\parallel}$, wall-directed flow $\frac{-u_x}{\sin \alpha} = u_{\parallel} + \frac{1}{B \tan \alpha} \frac{\partial \phi}{\partial y}$, and $(c_S^*)^2$



REFLECTING BOUNDARY CONDITION: MASS AND MOMENTUM (IONS) PROPAGATE BACK FROM THE MAGNETIC PRESHEATH

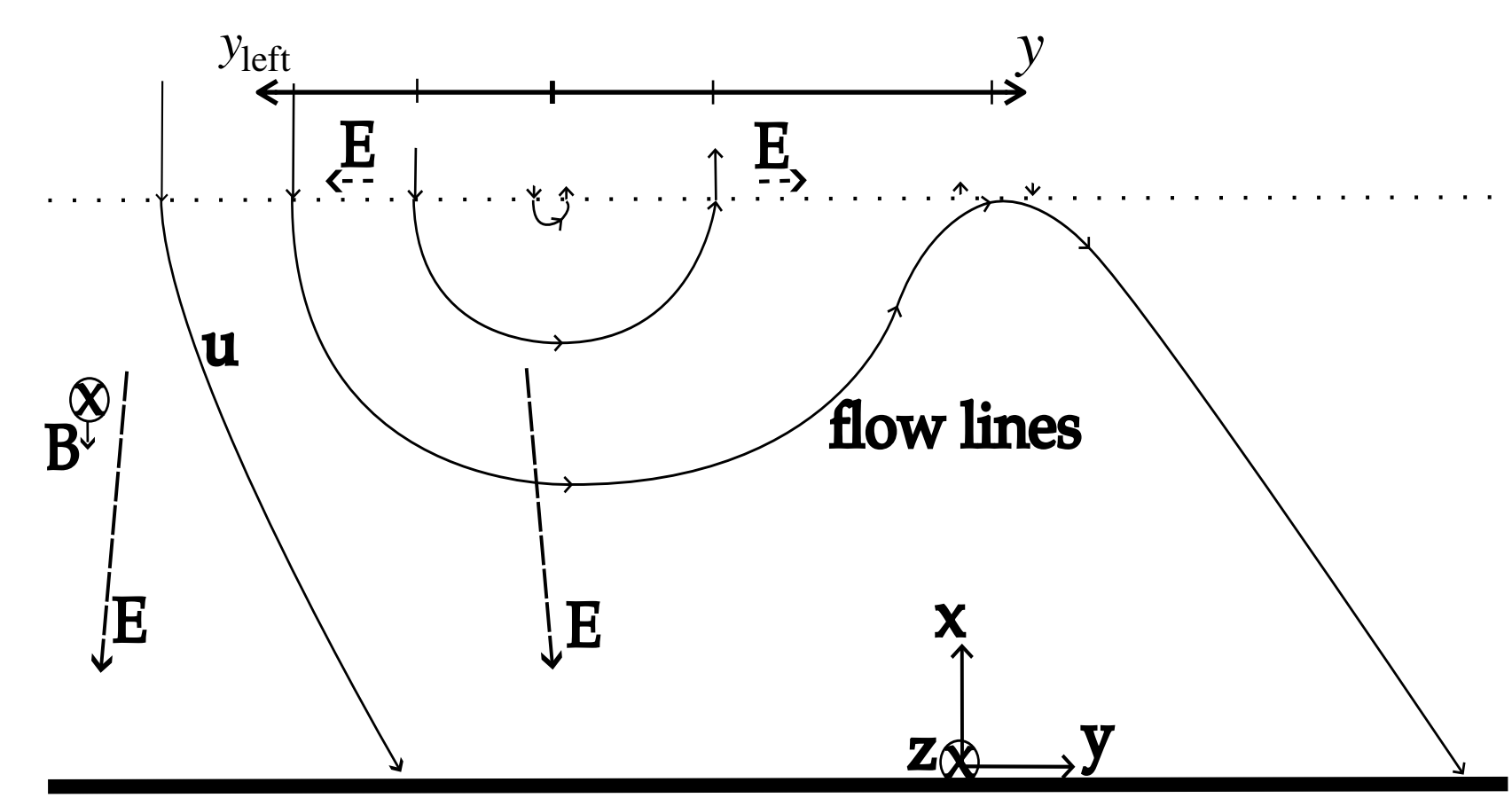
- Magnetic presheath electric field pushes ions to wall to reflect electrons
 —> causes ExB drift in +y direction —> causes ion to drift across perturbation
 —> ExB drift away from wall can increase —> ions can reflect away from wall

- Flow conserves mass and canonical parallel momentum [4]

$$\frac{dy_{\text{left}}}{dy} = \frac{n(y) u_x(y)}{n(y_{\text{left}}) u_x(y_{\text{left}})}$$

$$\frac{u_{\parallel}(y) - u_{\parallel}(y_{\text{left}})}{\Omega \tan \alpha} = y - y_{\text{left}}$$

- Integrate in +y => reflecting b.c.



STABILITY CONSIDERATIONS

- Kelvin-Helmholtz (KH) instability when c_S^* is imaginary [5]

$$\frac{1}{\Omega} \frac{\partial u_{\parallel}}{\partial y} > \tan \alpha \left(1 + \frac{c_S^2}{4 \Omega^2 \tan^2 \alpha} \left(\frac{\partial \ln n_e}{\partial y} \right)^2 \right)$$

- Perturbative Bohm condition becomes KH unstable before reflection
- Bohm condition hardwires stability at the boundary, but can lead to reflection
- Reflecting condition becomes KH unstable

SUMMARY AND OUTLOOK

- Developing rigorous first-principles derivation of ion flow boundary condition with drifts pushing ions to/from wall
- Full condition involves $du_{\parallel}/dy$ and is therefore a differential equation for the parallel ion flow w.r.t. y , rather than an explicit equation for the flow
- Perturbative approximations can be wrong when $\delta/\tan \alpha \sim 1$
- When wall-directed flow reverses, self-consistent ion-reflecting condition must result from flow circulation in magnetic presheath (absorbing wall!)
- Here found $u_{\parallel}$ given $\ln n$ (and ϕ), in reality they are intertwined

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